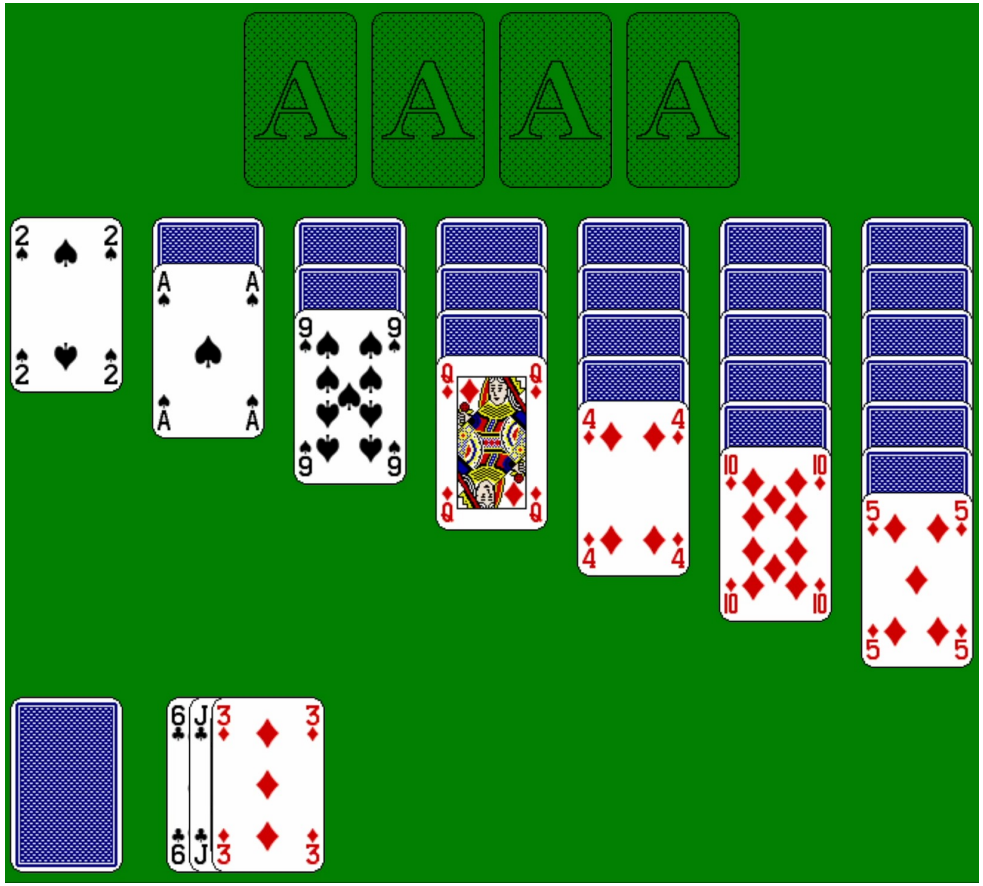
Solving Patience and Solitaire Games with Good Old Fashioned AI

Ian Gent
University of St Andrews
Scotland, UK

https://ian.gent



Fun Fact!

My URL really is

https://ian.gent/

Click on the game image for page of links like slides, papers, etc

Ian Gent

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Patience and Solitaire Games

Solvitaire solver for many patience and solitaire games

Page of links related to Patience and Solitaire including slides and copies of papers from my CP2024 talk.

External Activities

Depressed Academics a blog for and by academics with depression, co-founded by me and Mikael Vejdemo-Johansson in 2013.

My personal blog, most noted for my blog post about the Petrie Multiplier, invented by Karen Petrie but named by me.

Based on two papers

arXiv:1906.12314 (Version 5) August 2024 https://arxiv.org/abs/1906.12314v5

The Winnability of Klondike Solitaire and Many Other Patience Games

Charlie Blake THECHARLIEBLAKE@GMAIL.COM
Work undertaken while at School of Computer Science, University of St Andrews, St Andrews, UK
Ian P. Gent IAN.GENT@ST-ANDREWS.AC.UK
School of Computer Science, University of St Andrews, St Andrews, UK (Corresponding Author)

About to be resubmitted to JAIR

Fun Fact

- The original submission was rejected from JAIR
- But had some amazingly kind comments

“I also enjoyed that this paper has soul … they formatted it in the way they thought would be best for their work. ”

Based on two papers

Constraint Models for Relaxed Klondike Variants

Nguyen Dang
School of Computer Science, University of St Andrews, UK

Ian P. Gent
School of Computer Science, University of St Andrews, UK

Peter Nightingale
Department of Computer Science, University of York, UK

Felix Ulrich-Oltean
Department of Computer Science, University of York, UK

Jack Waller
School of Computer Science, University of St Andrews, UK

ModRef 2024

Joint Work With …

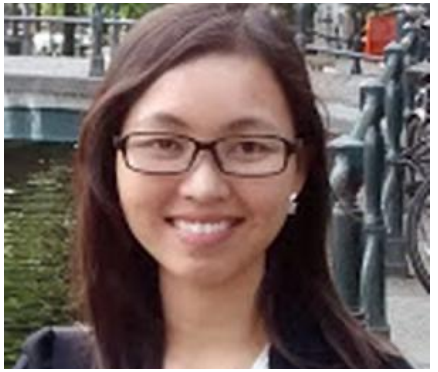
Nguyen Dang

Charlie Blake

Peter Nightingale

Jack Waller

Felix Ulrich-Oltean



Acknowledgements

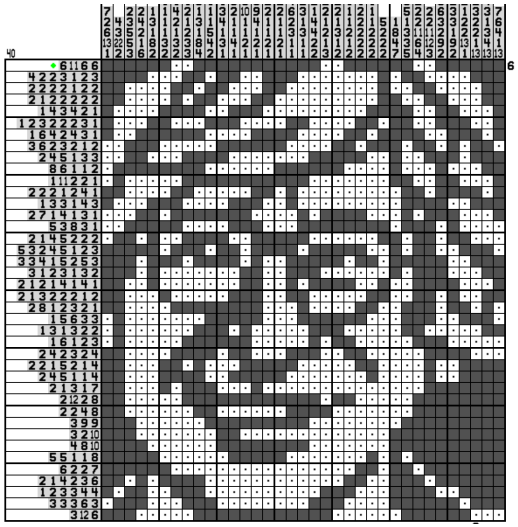
EPSRC for support

Cirrus UK National HPC Service for 30 cpu years

Mustafa Abdelwahed, Matt Birrell, Dawn Black, Laura Brewis, Arthur W. Cabral, Gal Cohensius, Joan Esparsa, Shlomi Fish, Jordina Francès de Mas, Ruth Hoffmann, Chris Jefferson, Michael Keller, Donald Knuth, Dana Mackenzie, Mark Masten, Ian Miguel, Theodore Pringle, Bill Roscoe, András Salamon, Judith Underwood

And Posthumously...

Jan Wolter, Hubert Gent, Margaret Gent



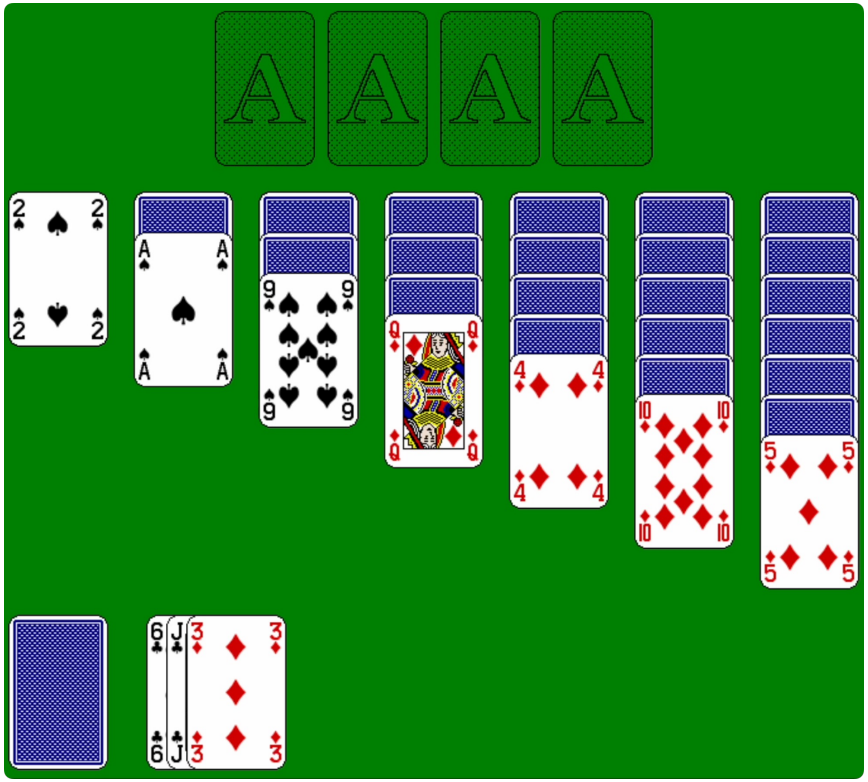
Conclusions

- The most famous single player card game is called “Klondike”
- Single player card games are called “Solitaire” or “Patience”
- There are hundreds of other Patiences
- We built a solver “Solvitaire” which can solve many many of these
- Solvitaire got world’s best results on dozens of Patience games
- Uses just a basic JSON description of rules
- It is a classic example of “Good Old Fashioned AI”
  - Depth-First Search with some GOFAI add-ons
  - Transposition Tables, Symmetries, Dominances, Streamliners
- And Solvitaire isn’t perfect so we can improve results with Constraints
- One huge open question remains

Conclusions

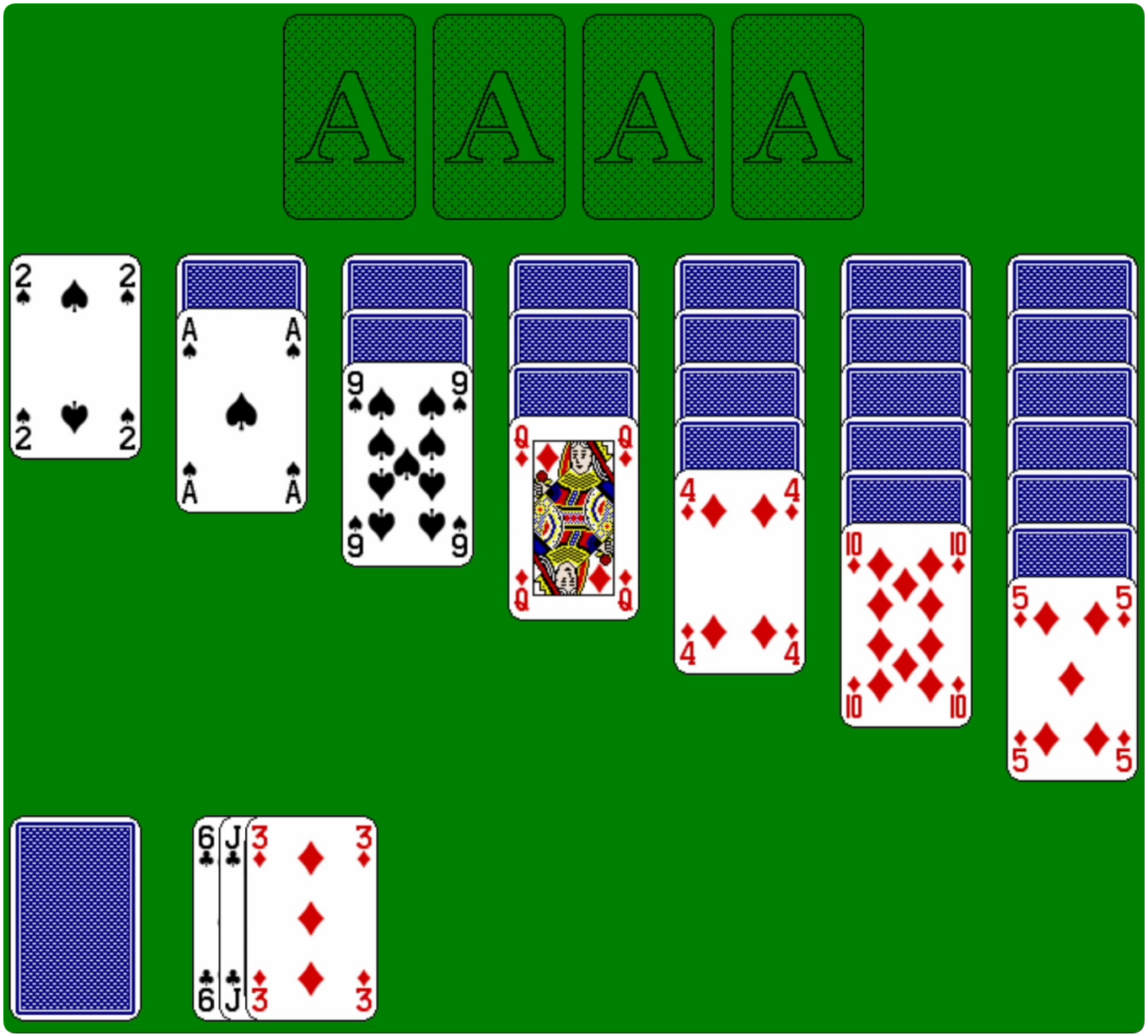
- The most famous single player card game is called “Klondike”

Klondike
the most famous Patience/Solitaire game



Fun Fact

- Microsoft Solitaire was getting 100 million daily plays in 2020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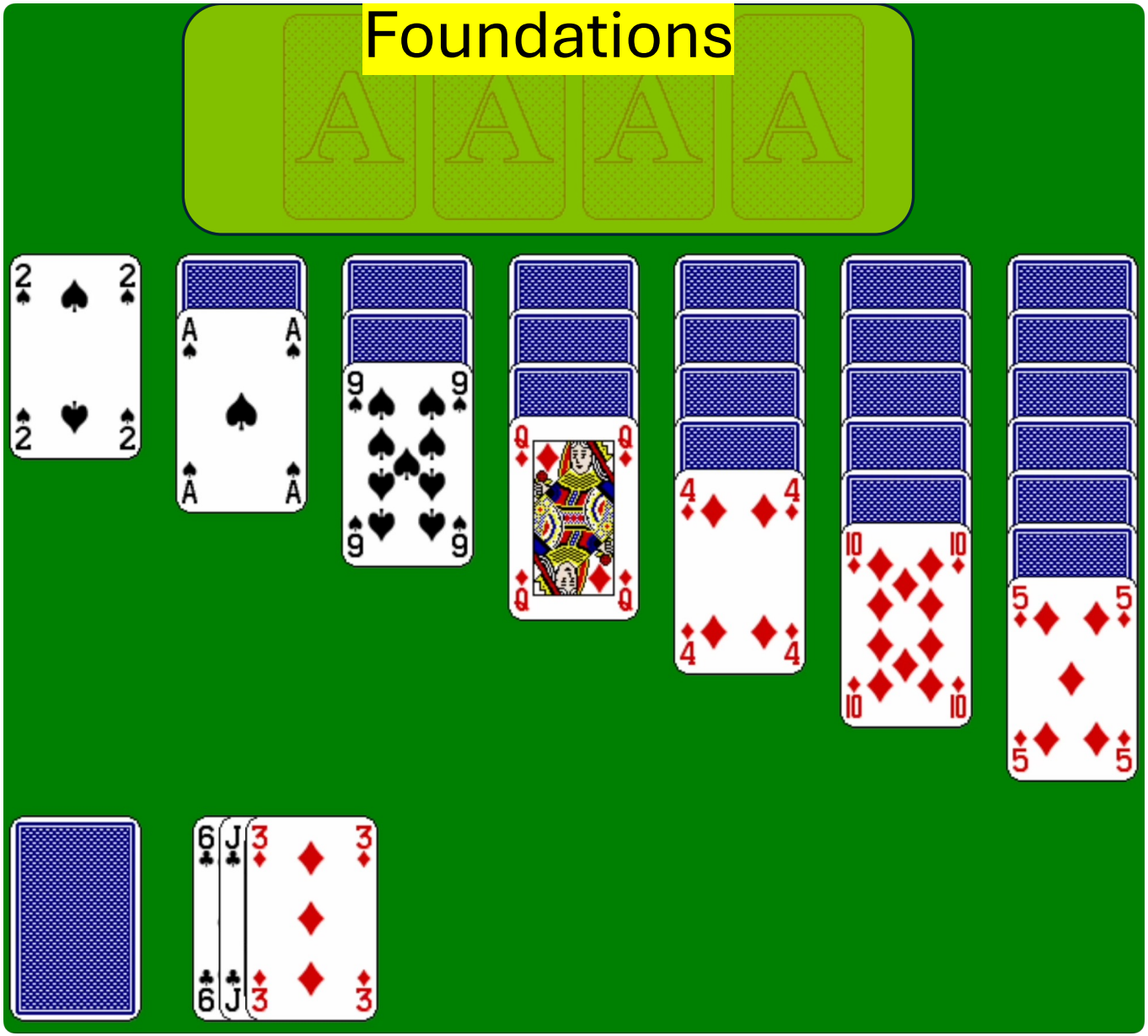
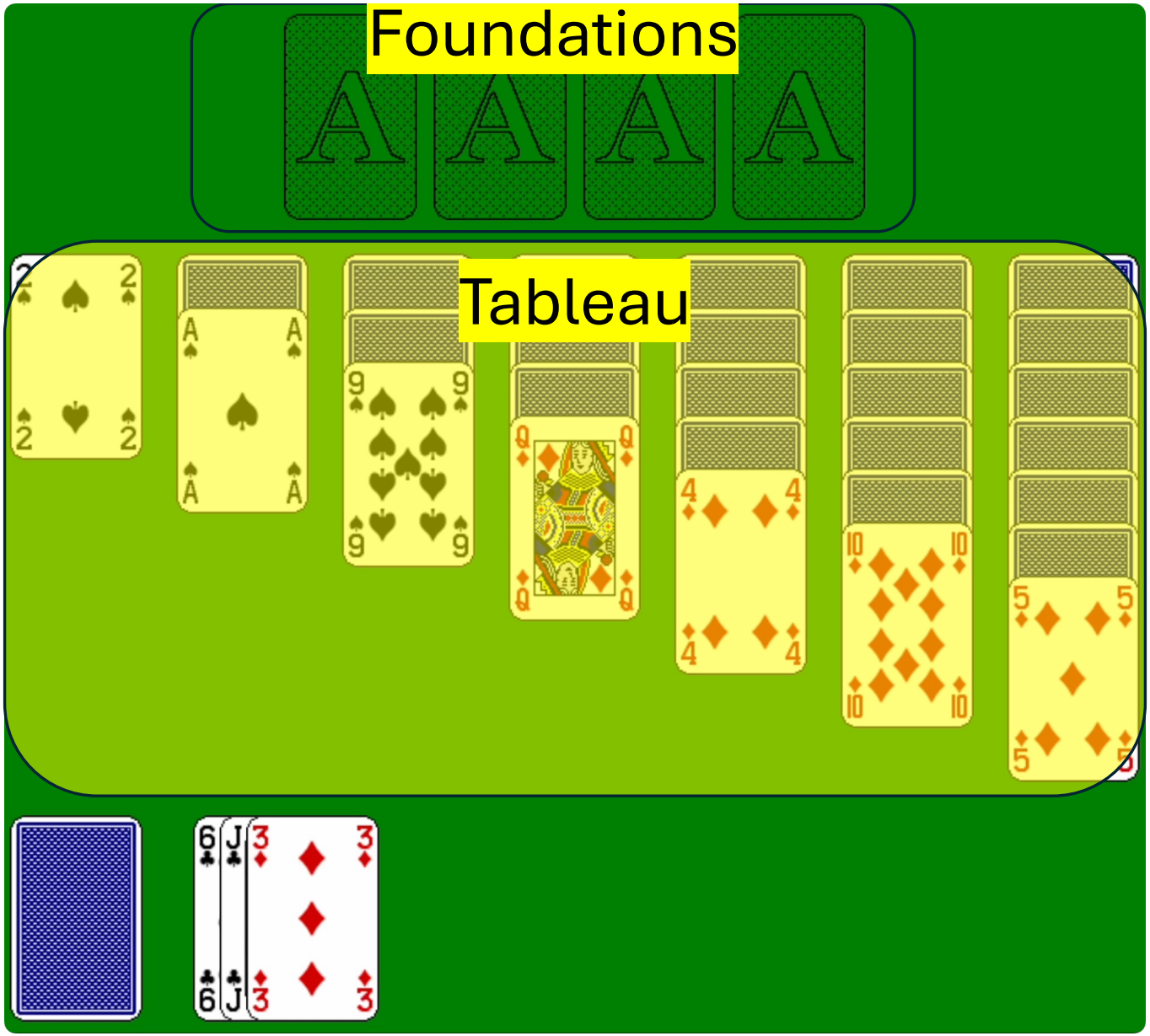
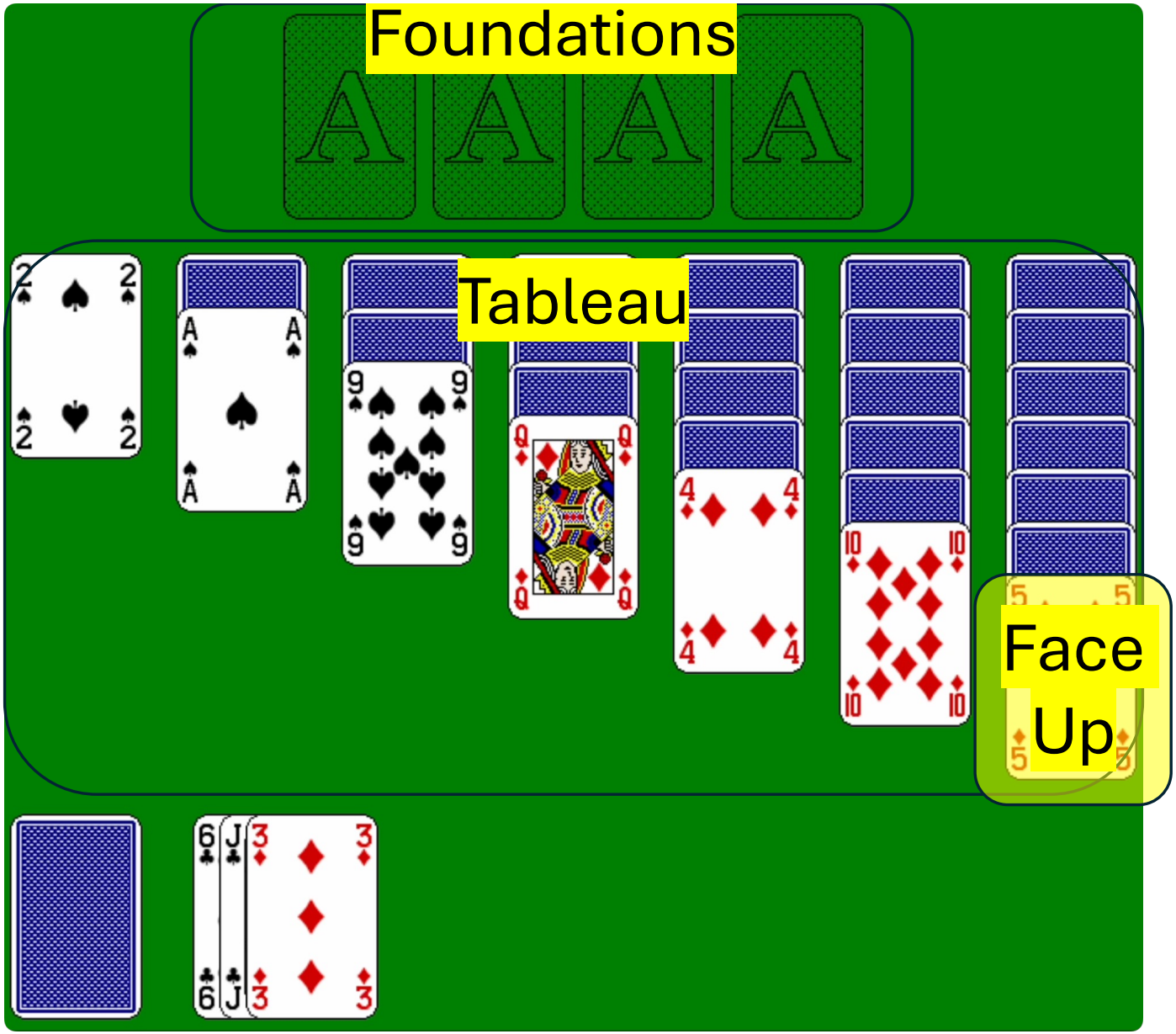
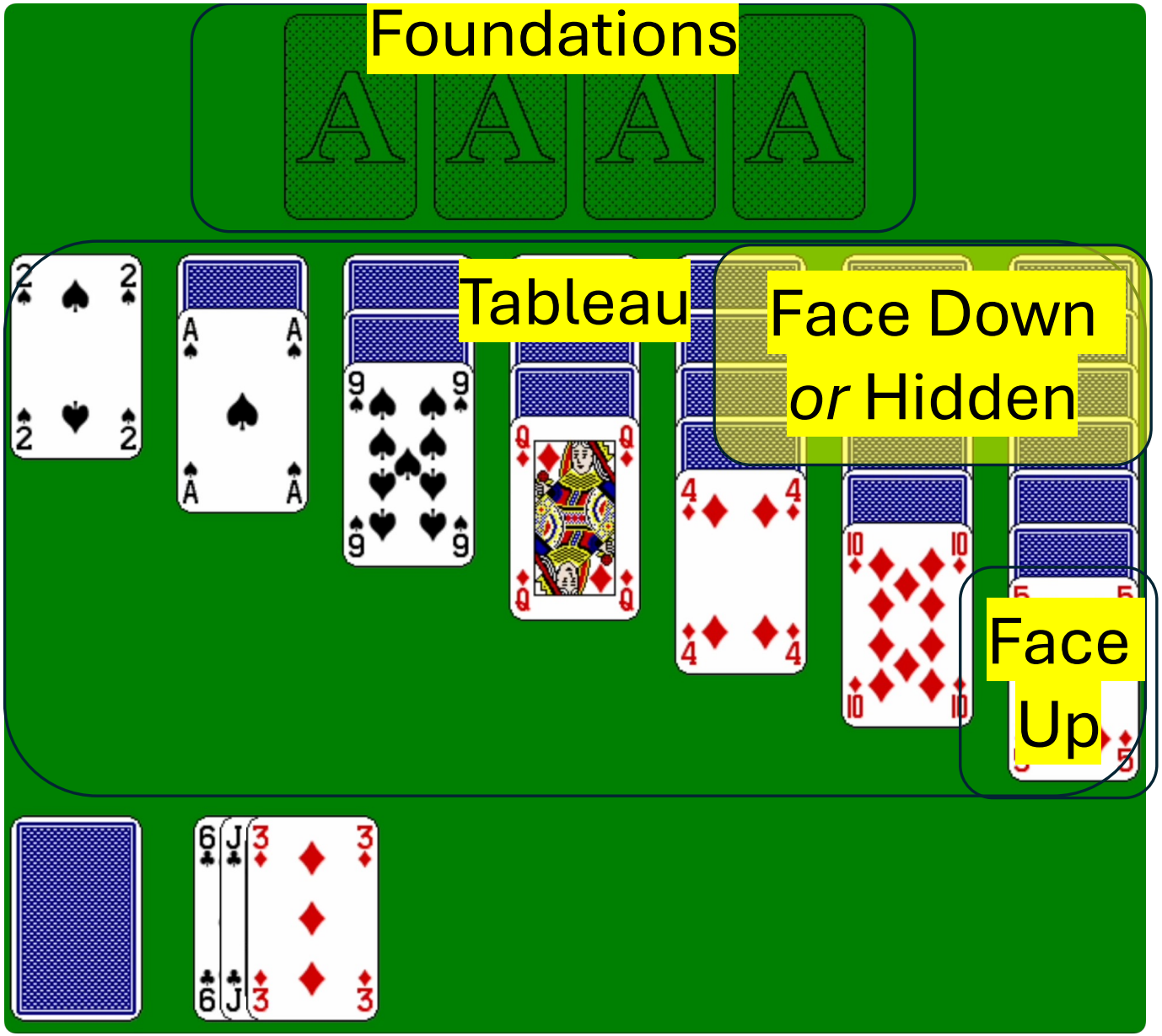
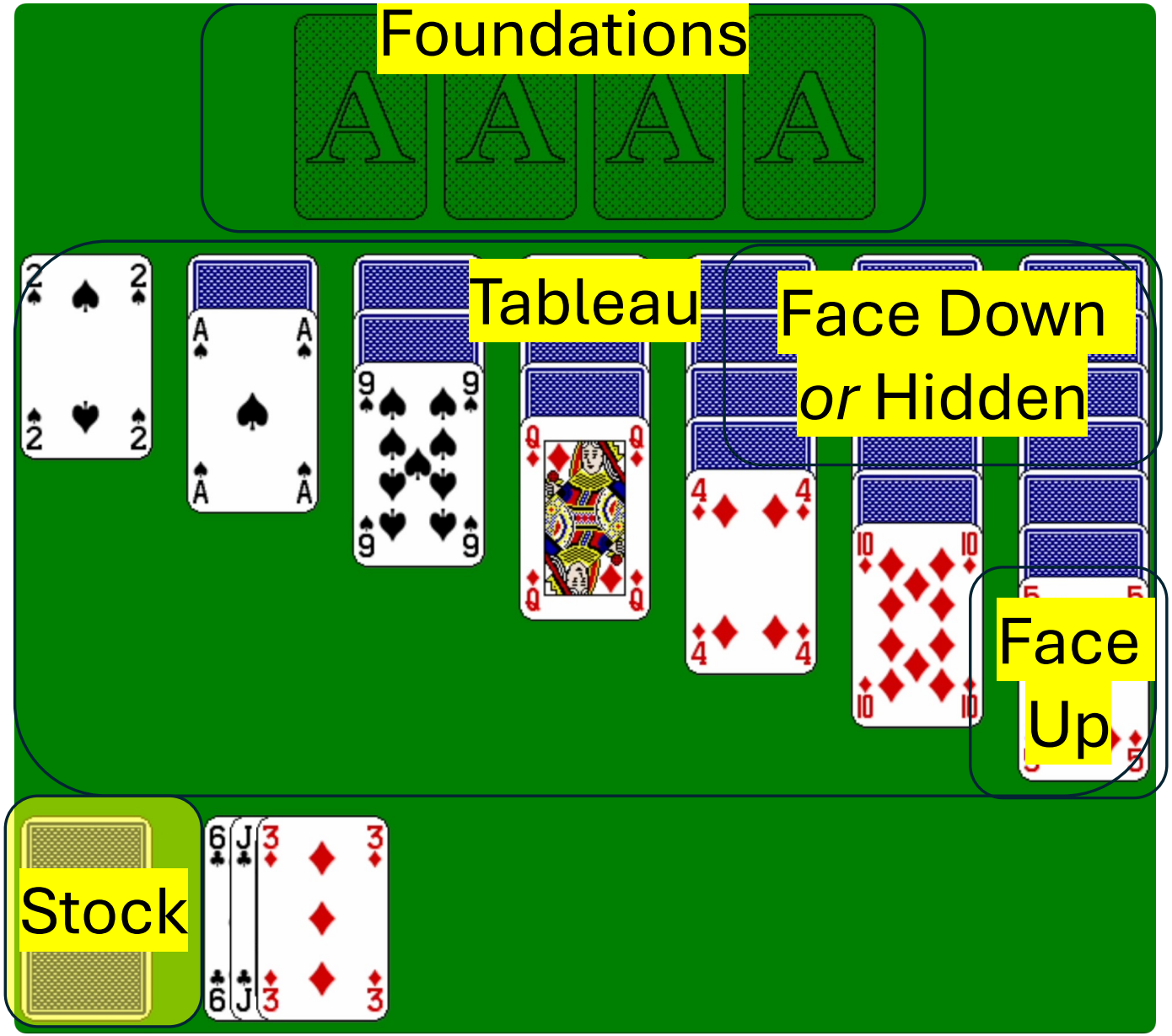
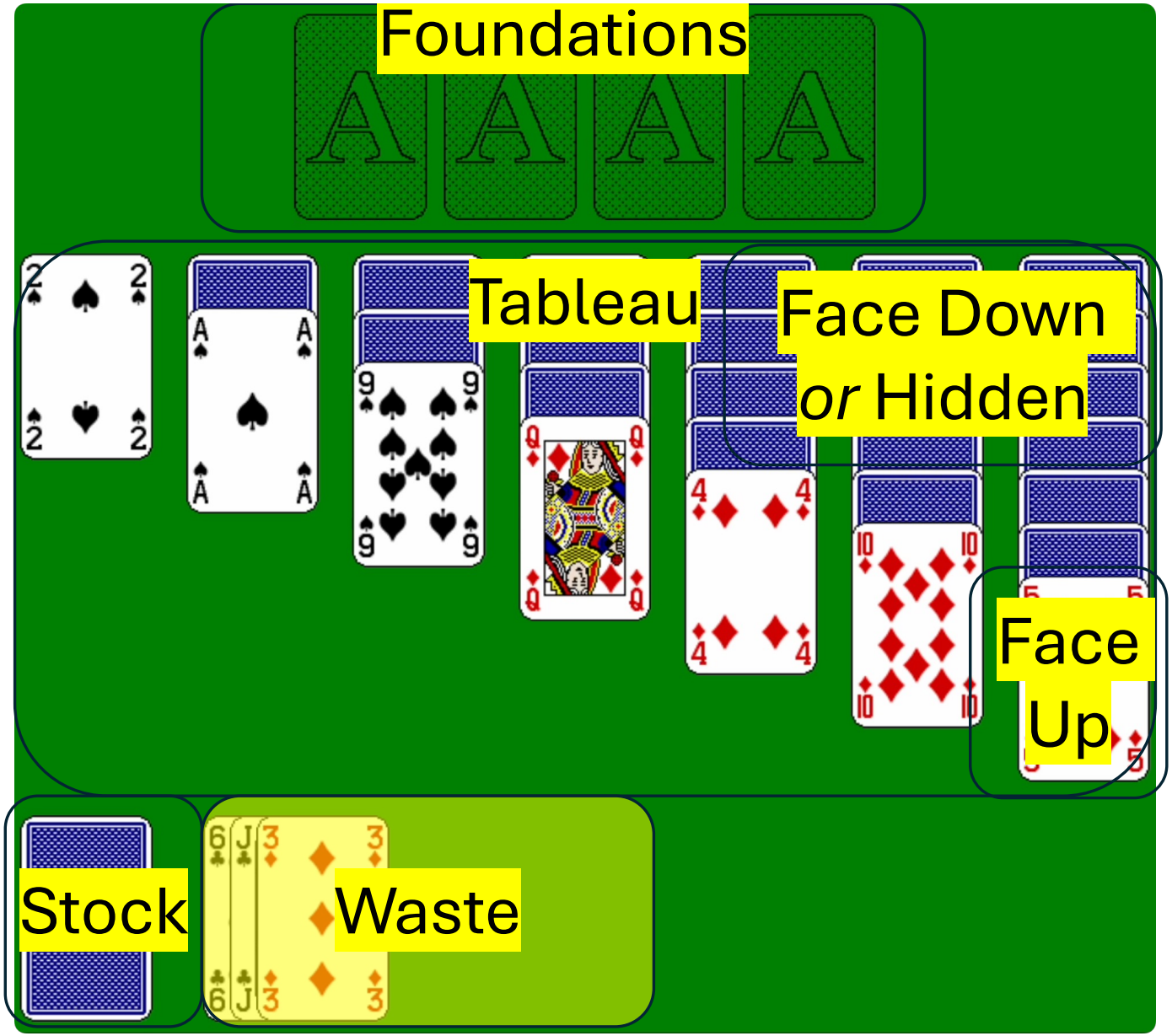

Foundations

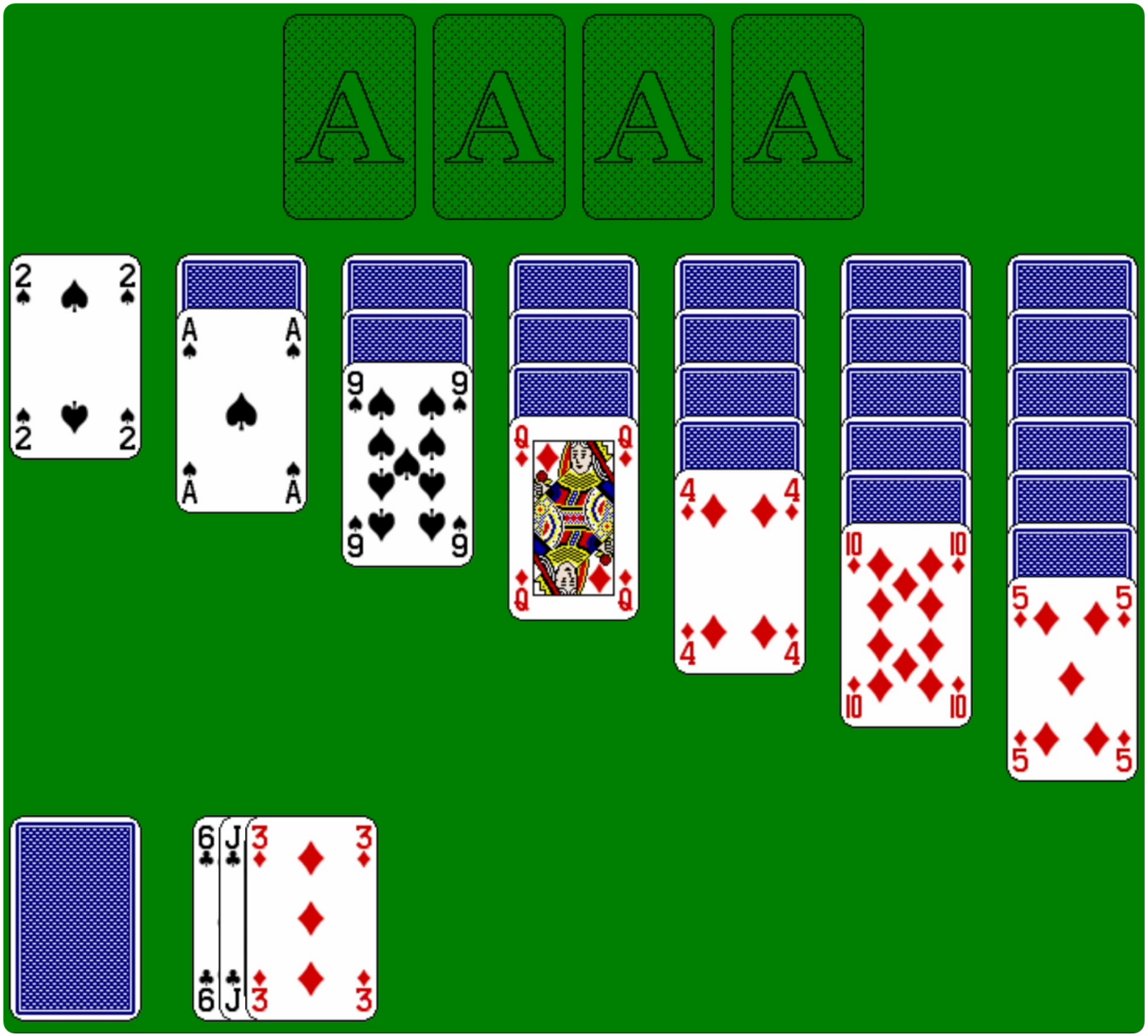

Foundations
Tableau


Foundations
Tableau
Face Up


Foundations
Tableau
Face Down
or Hidden
Face
Up


Foundations
Tableau
Face Down
or Hidden
Face
Up
Stock


Foundations
Tableau
Face Down
or Hidden
Face
Up
Stock
Waste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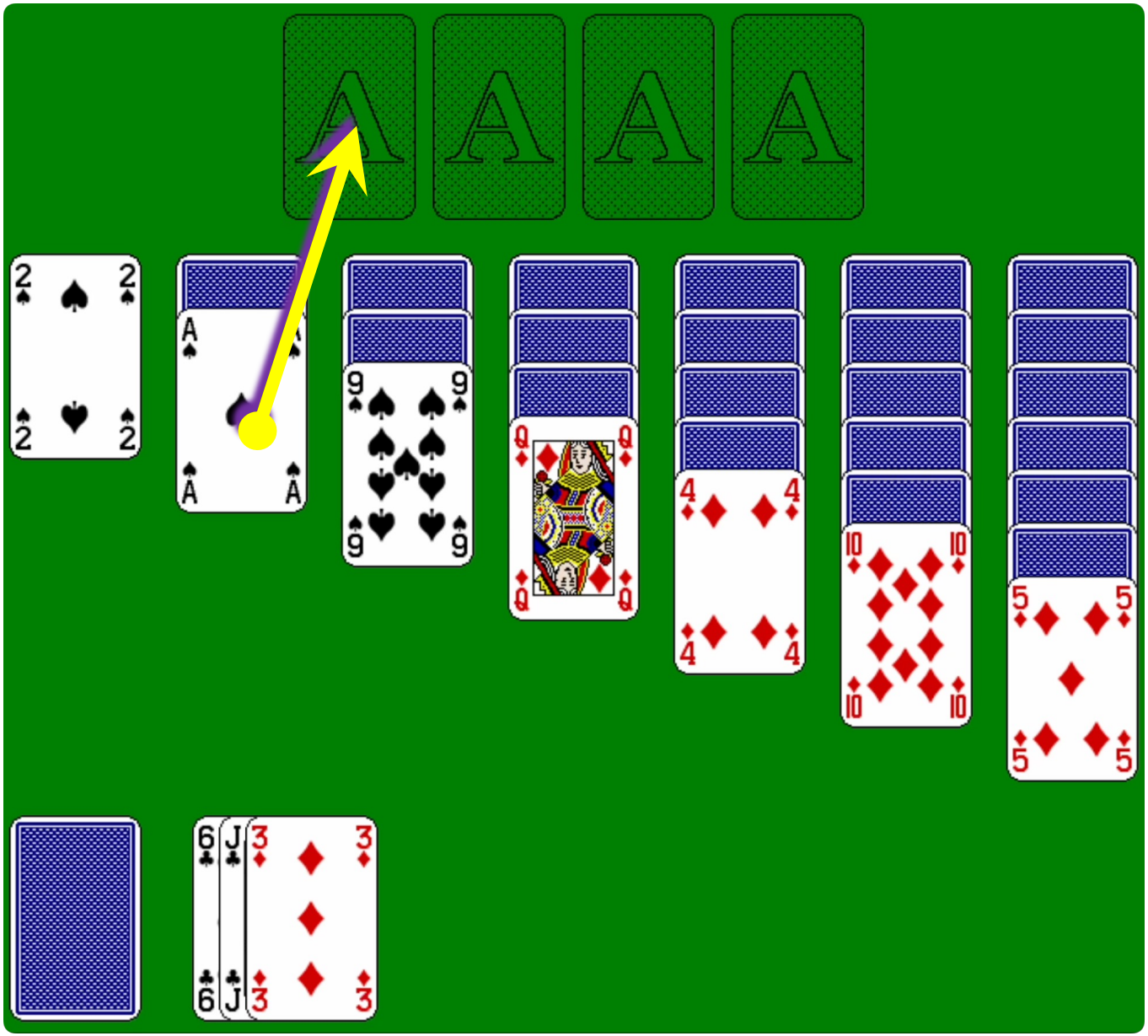
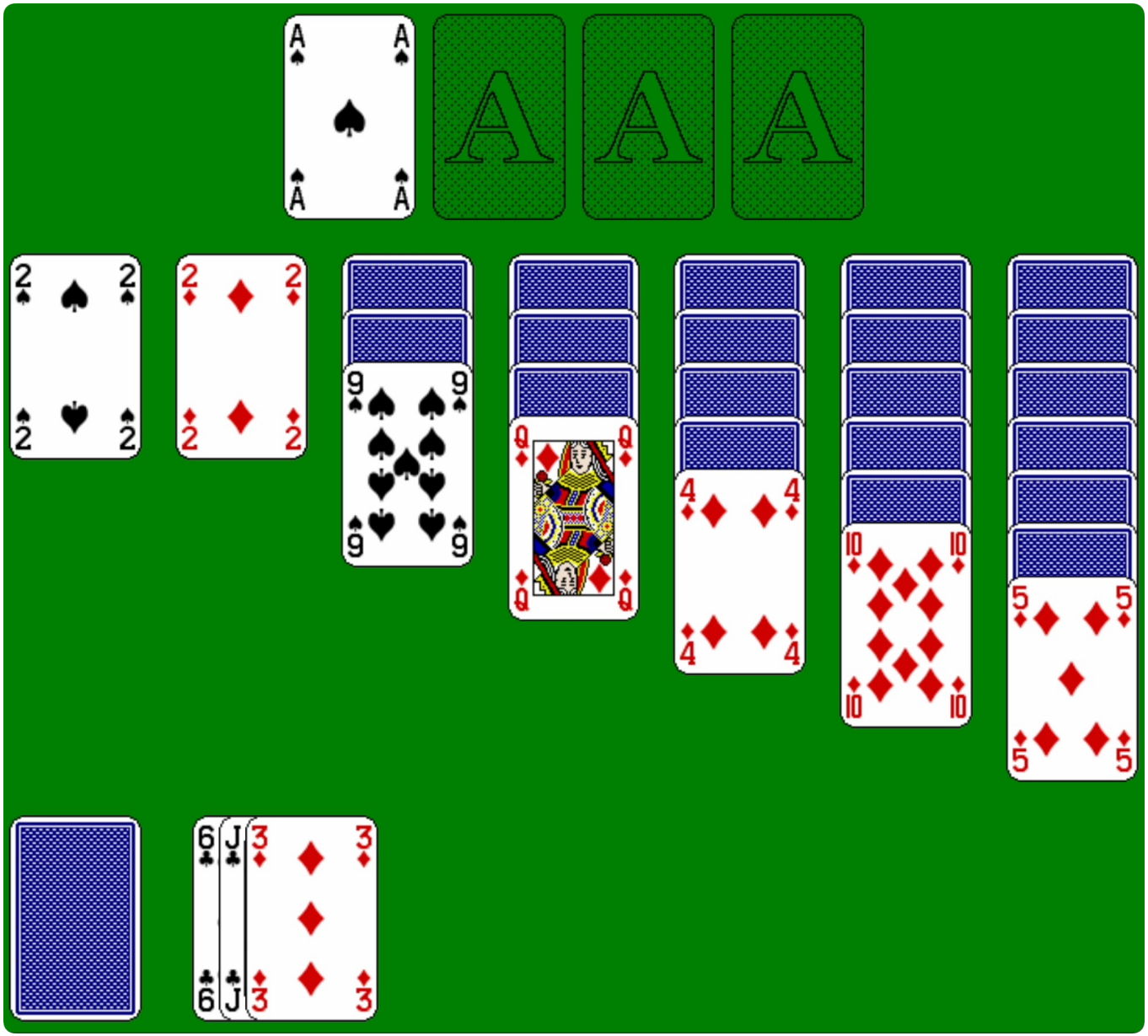
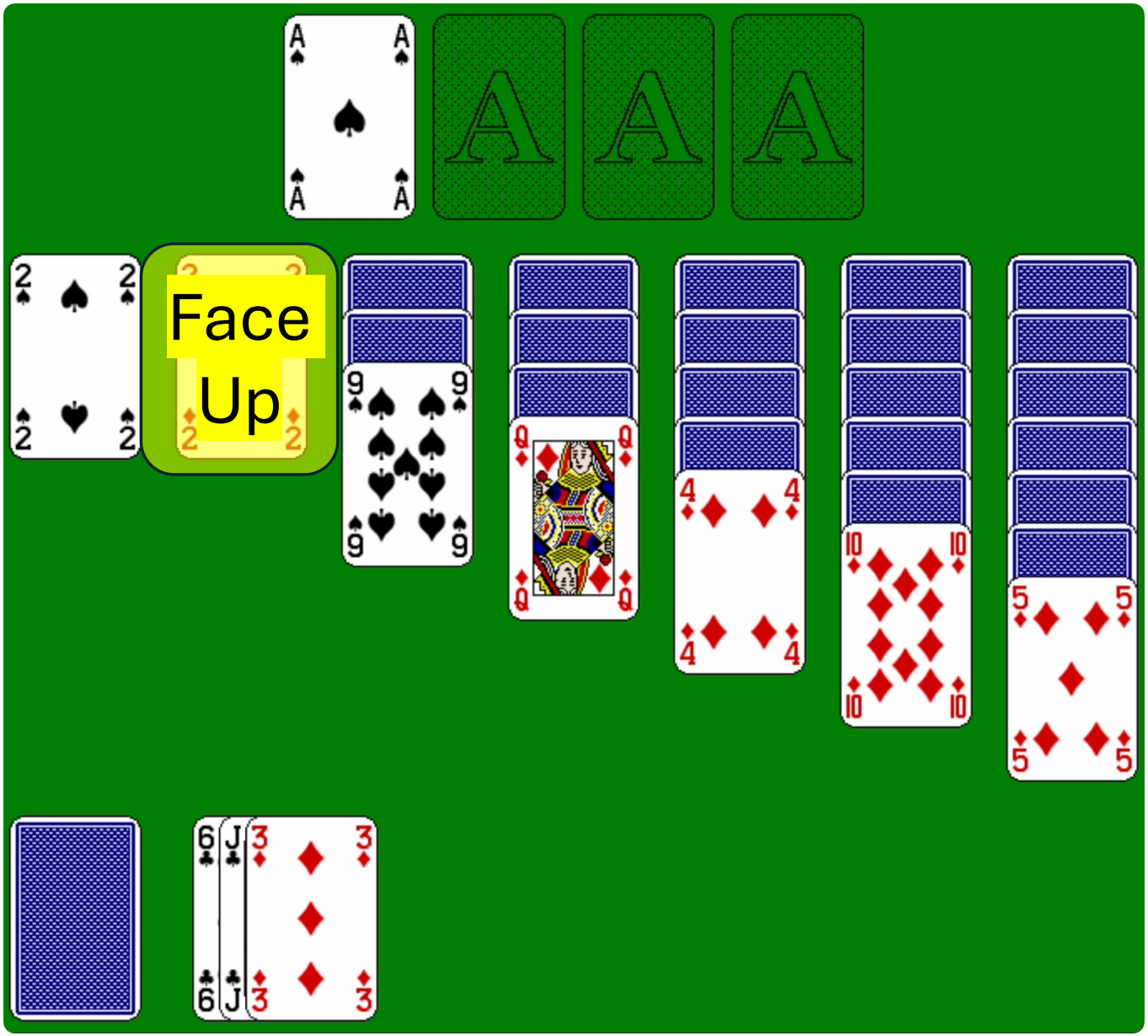
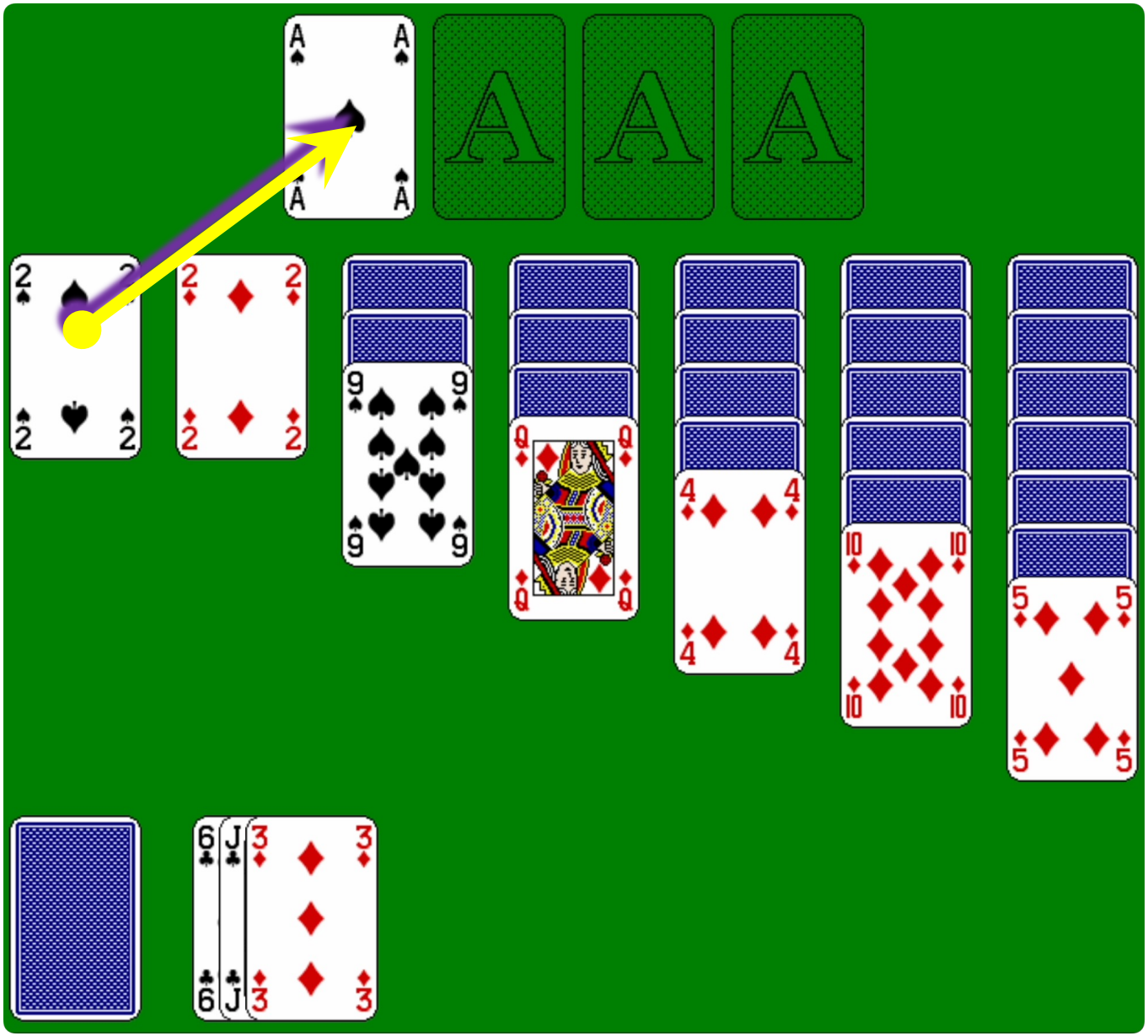
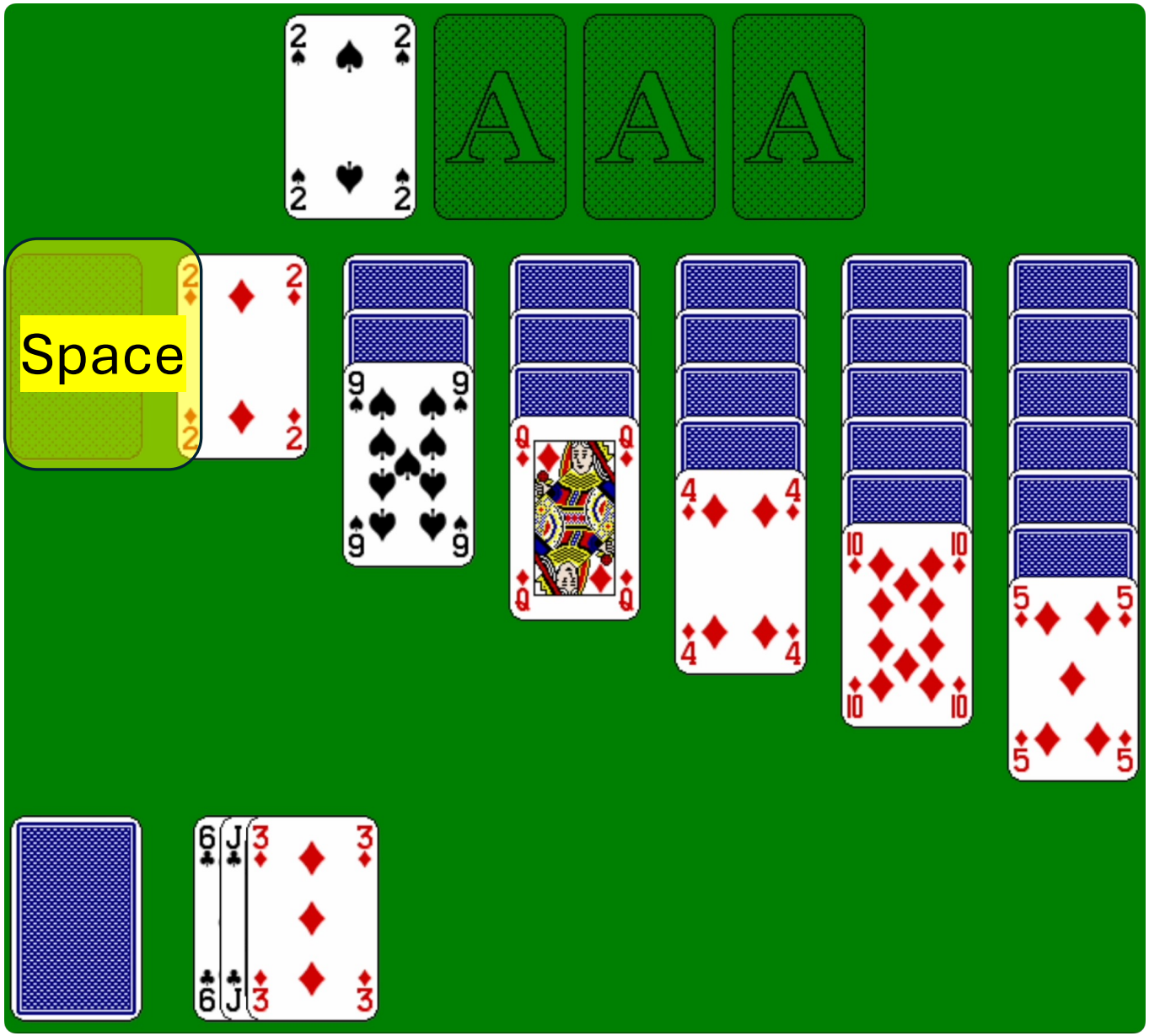





Face
Up

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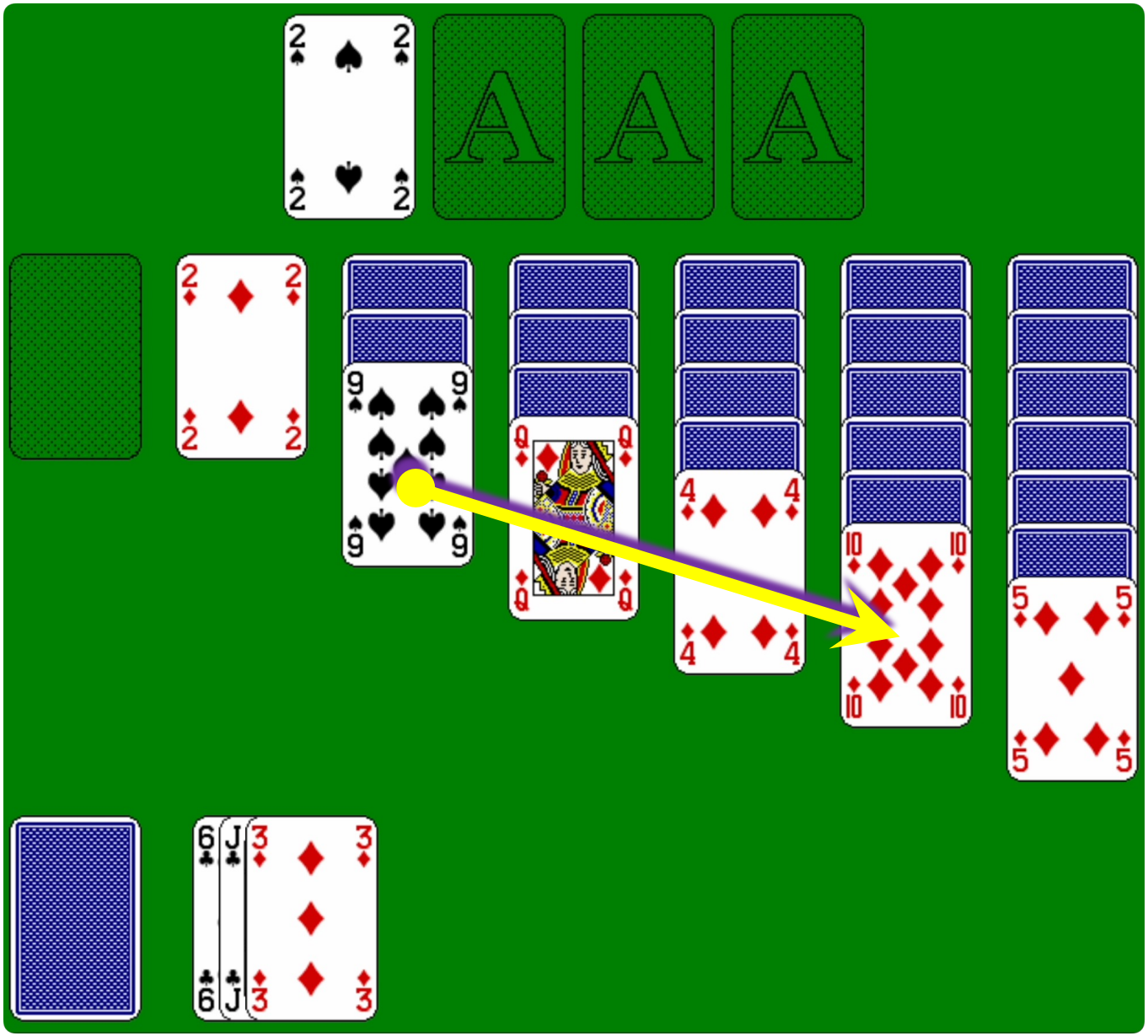
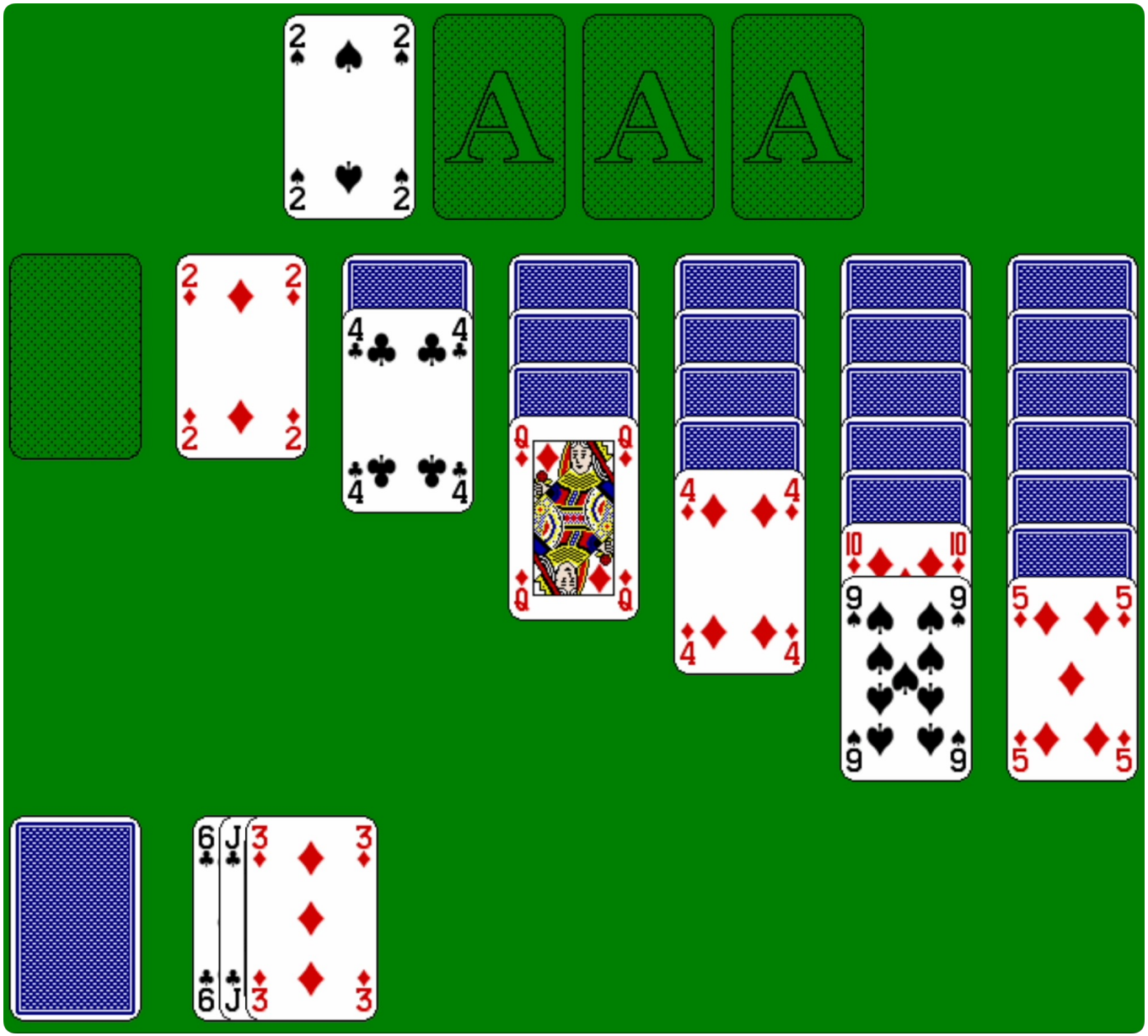
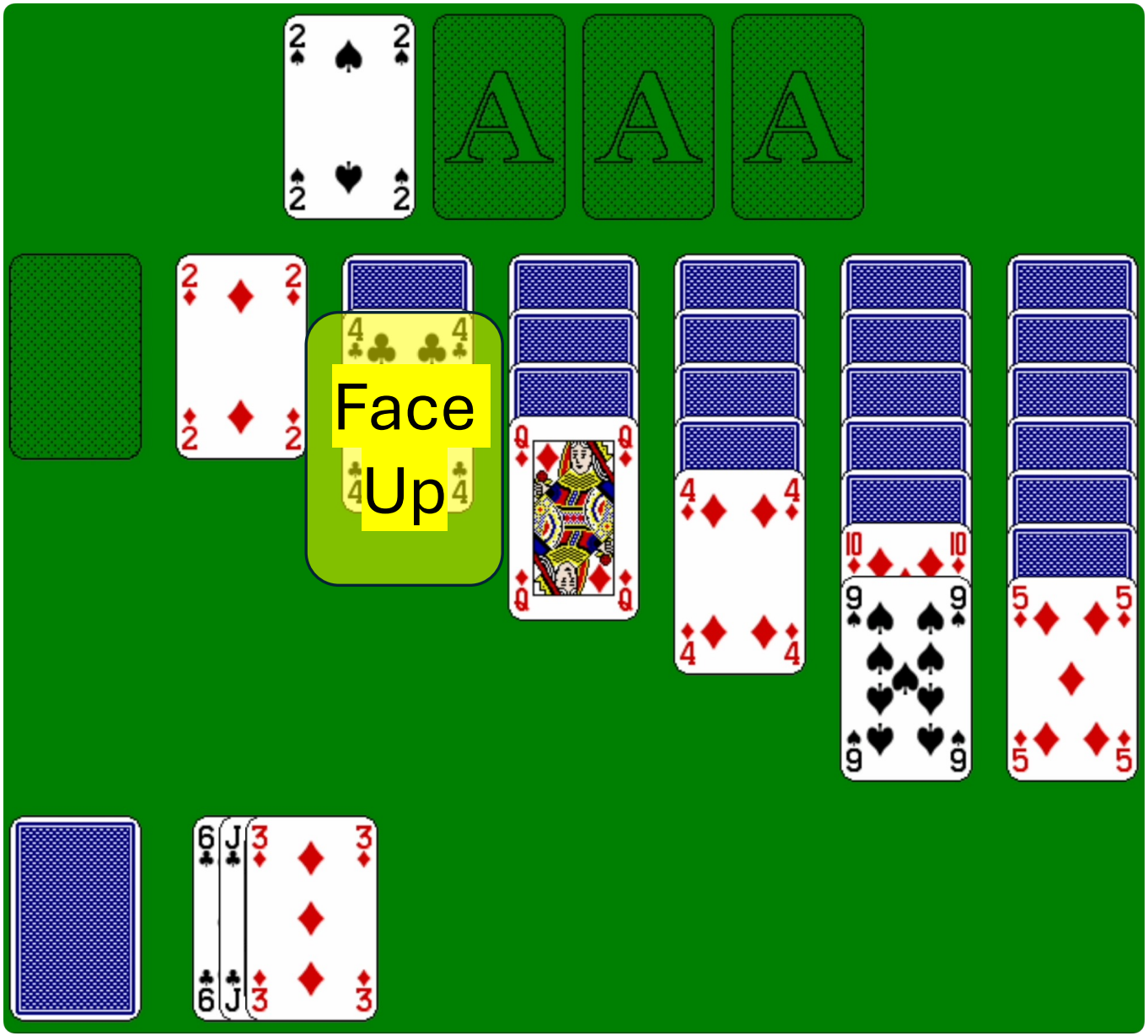
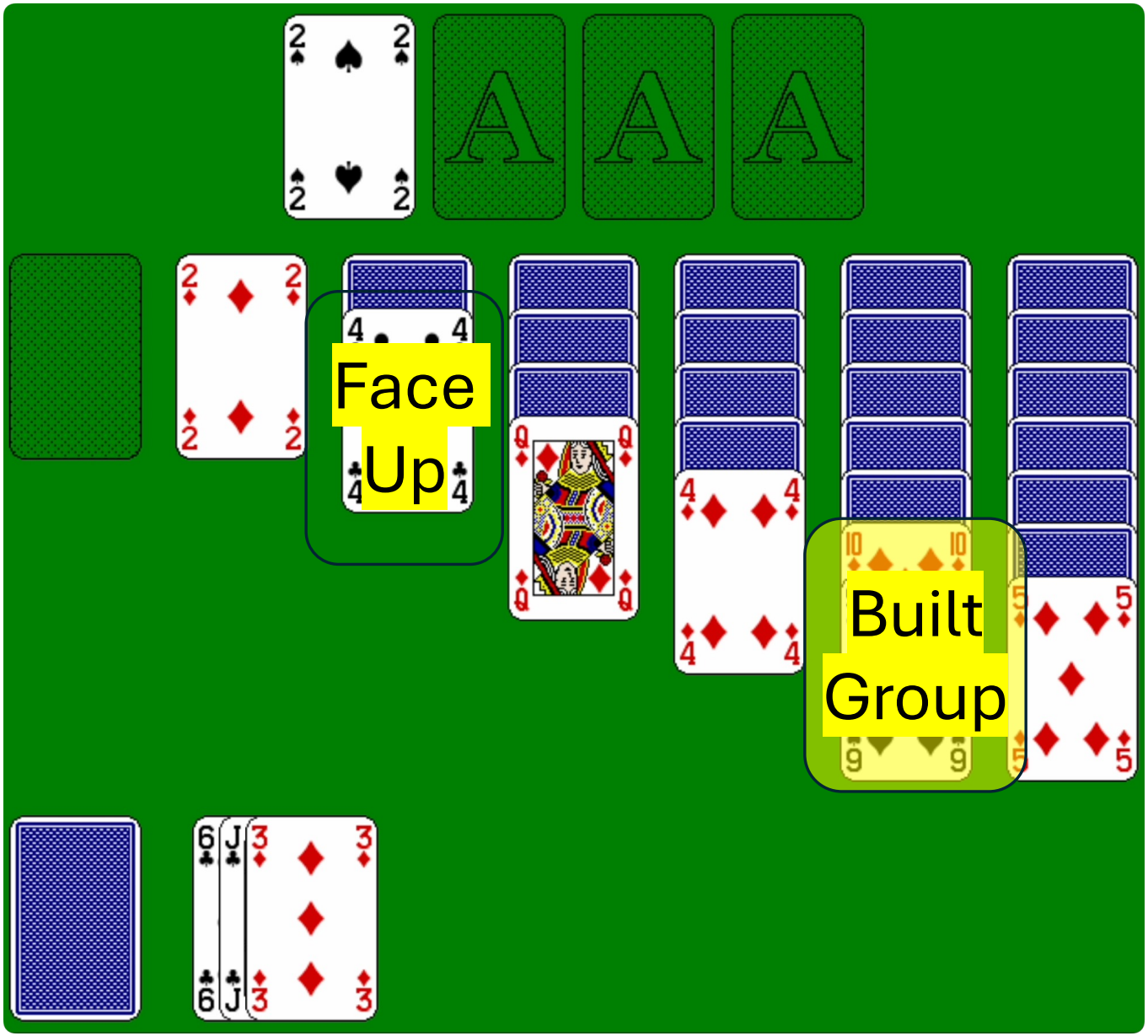
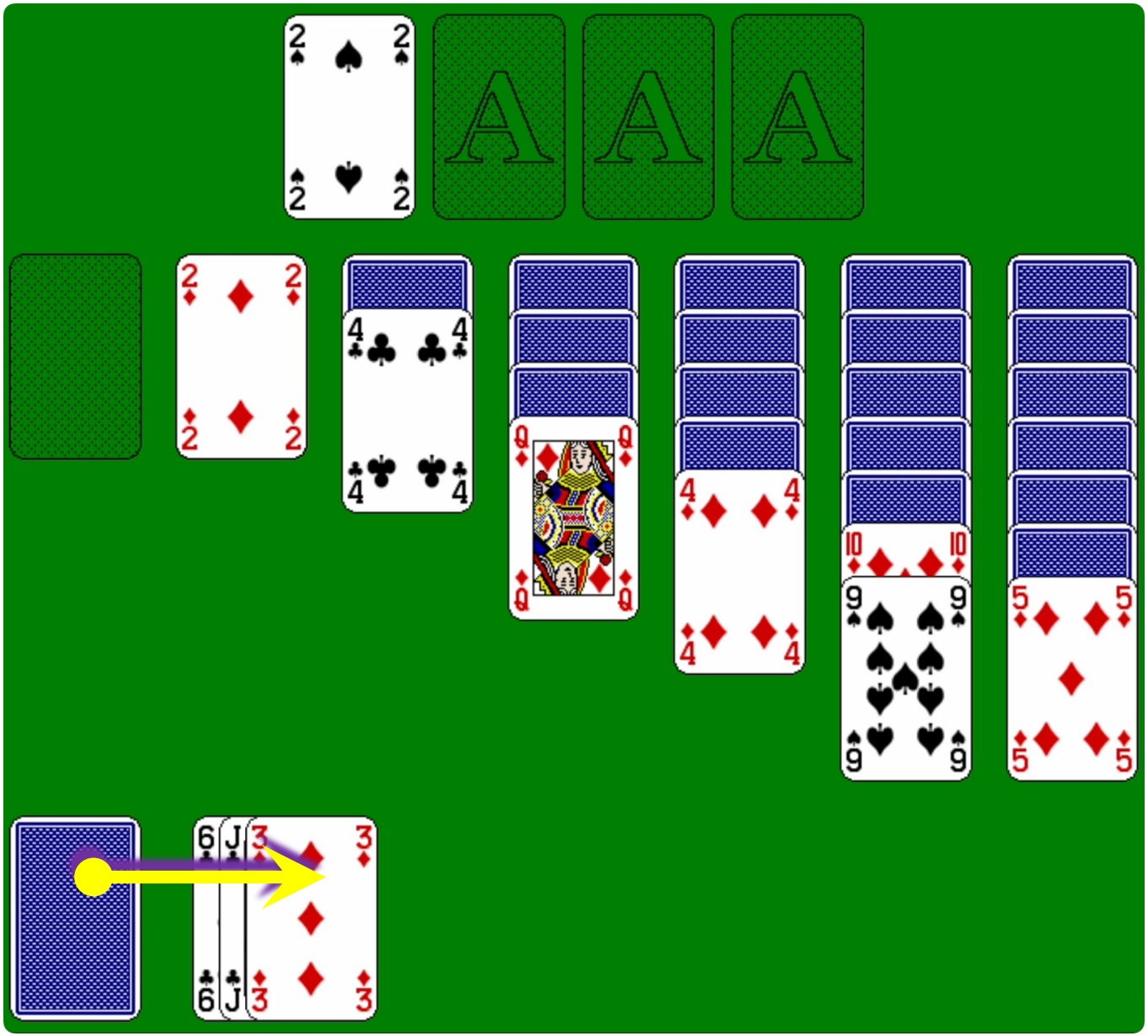
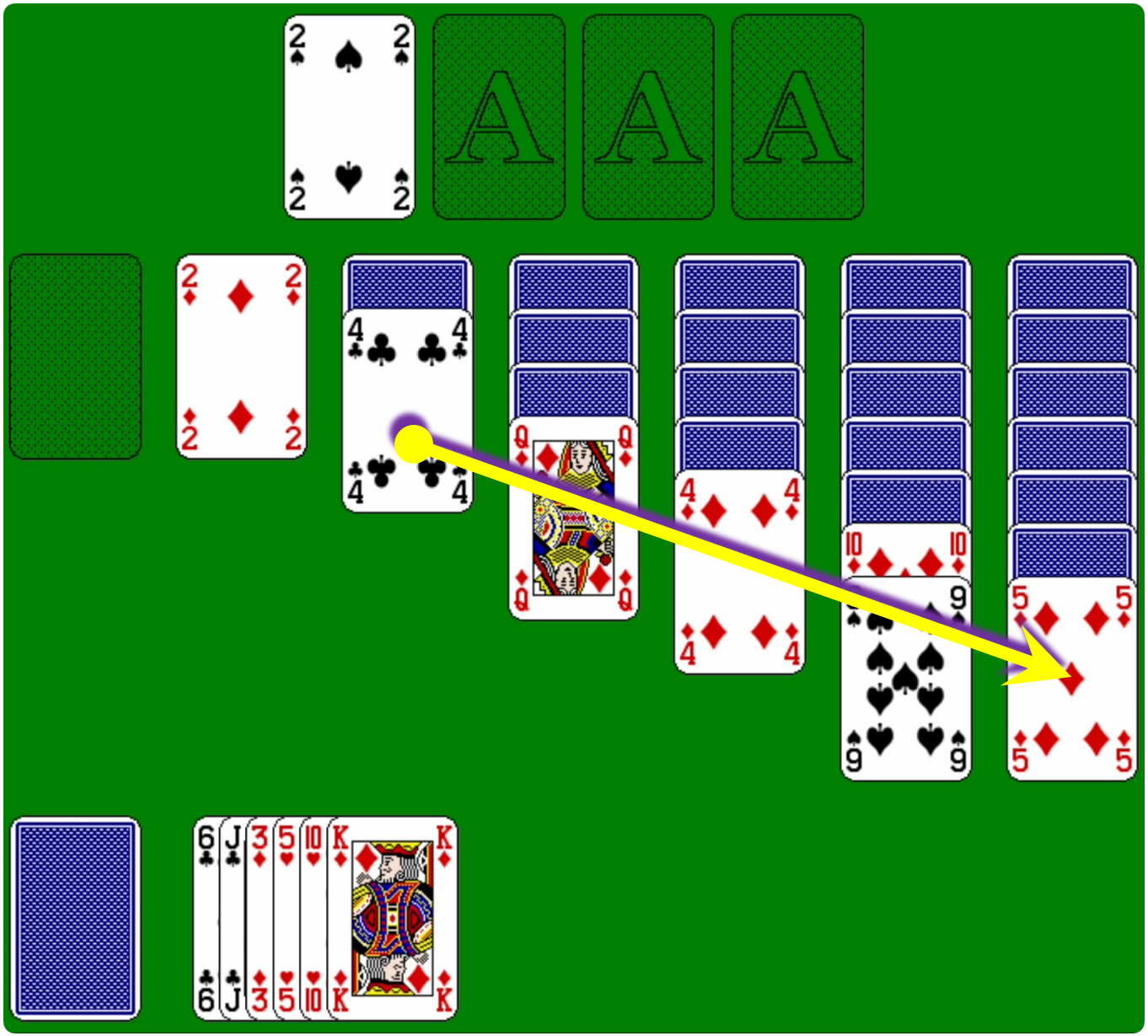


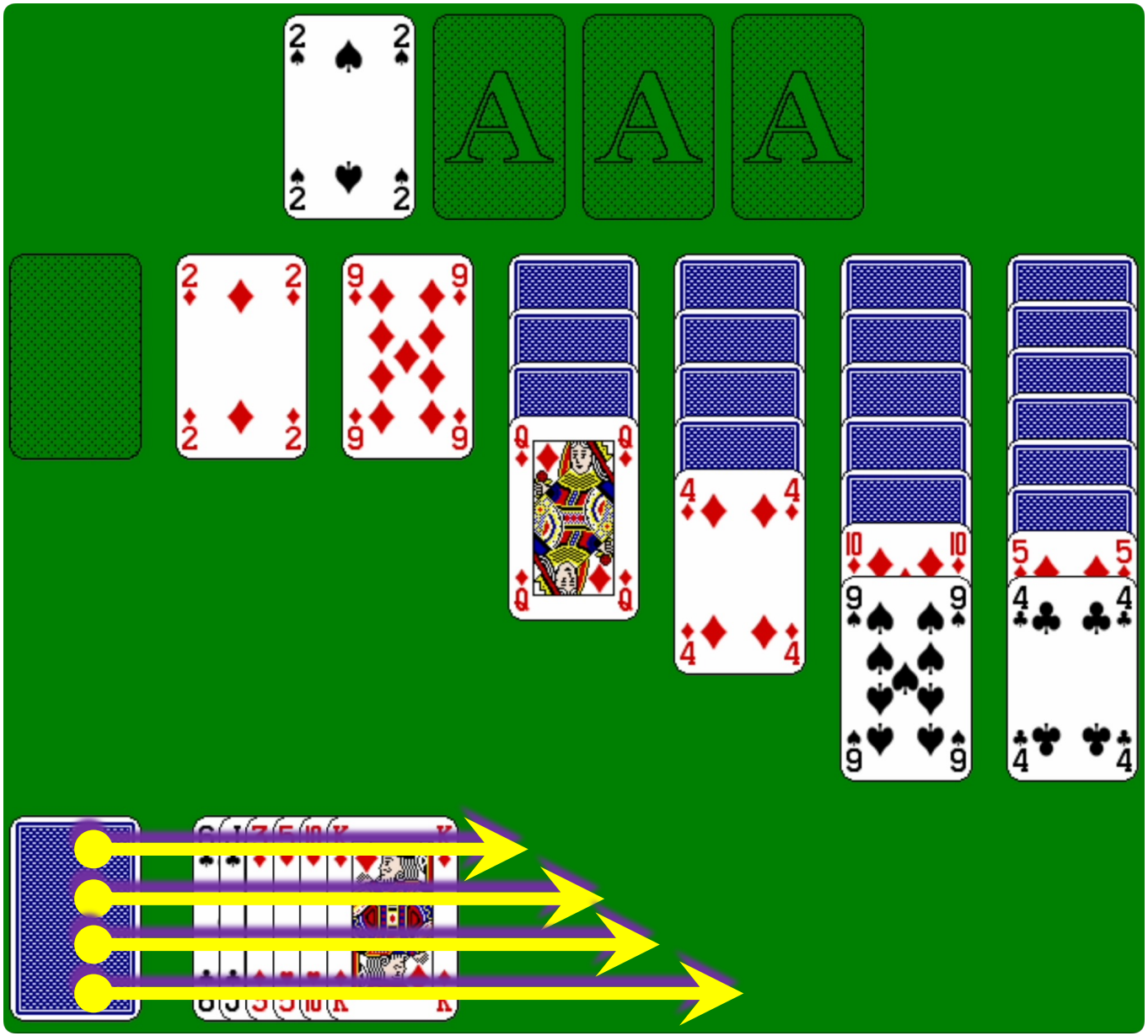


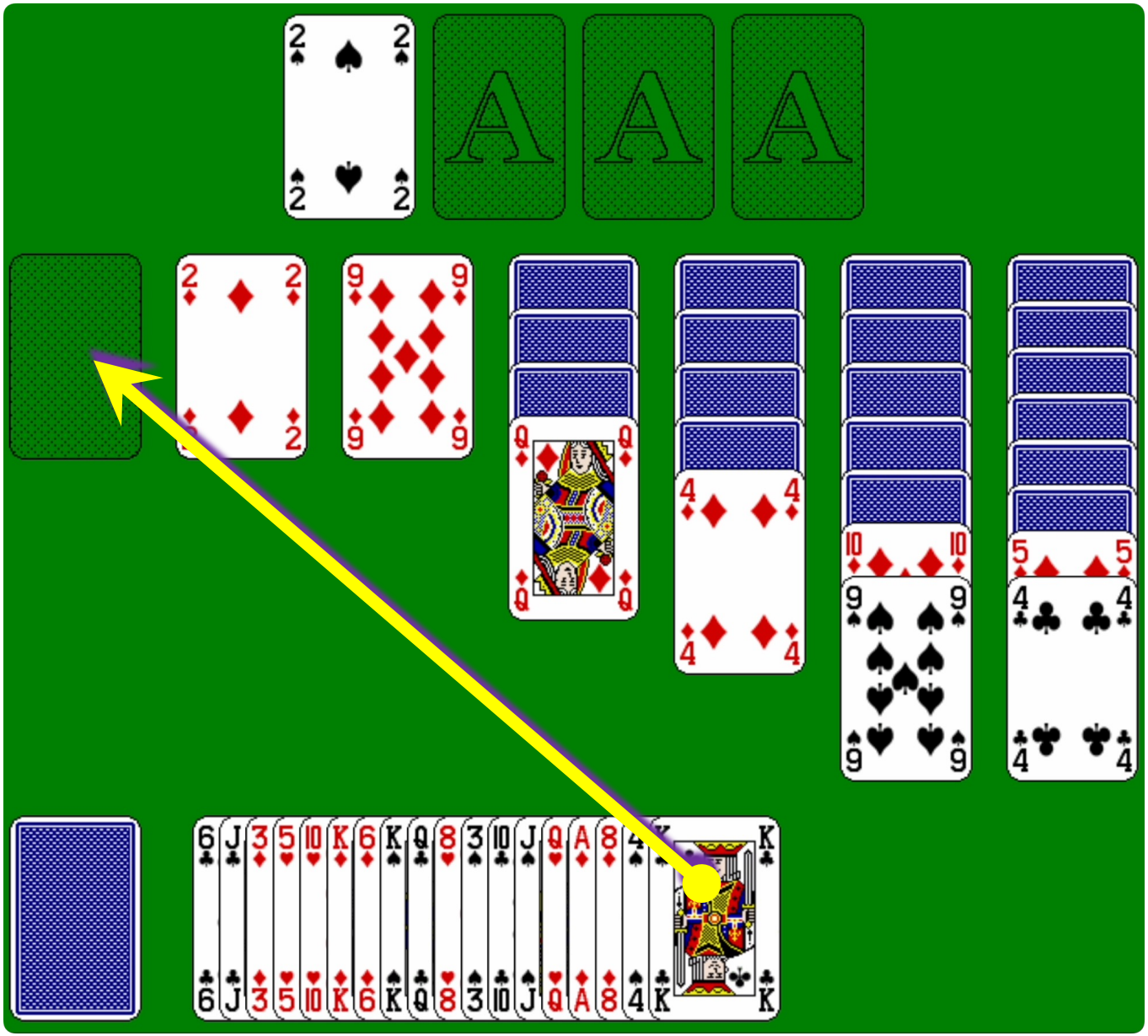

Face
Up

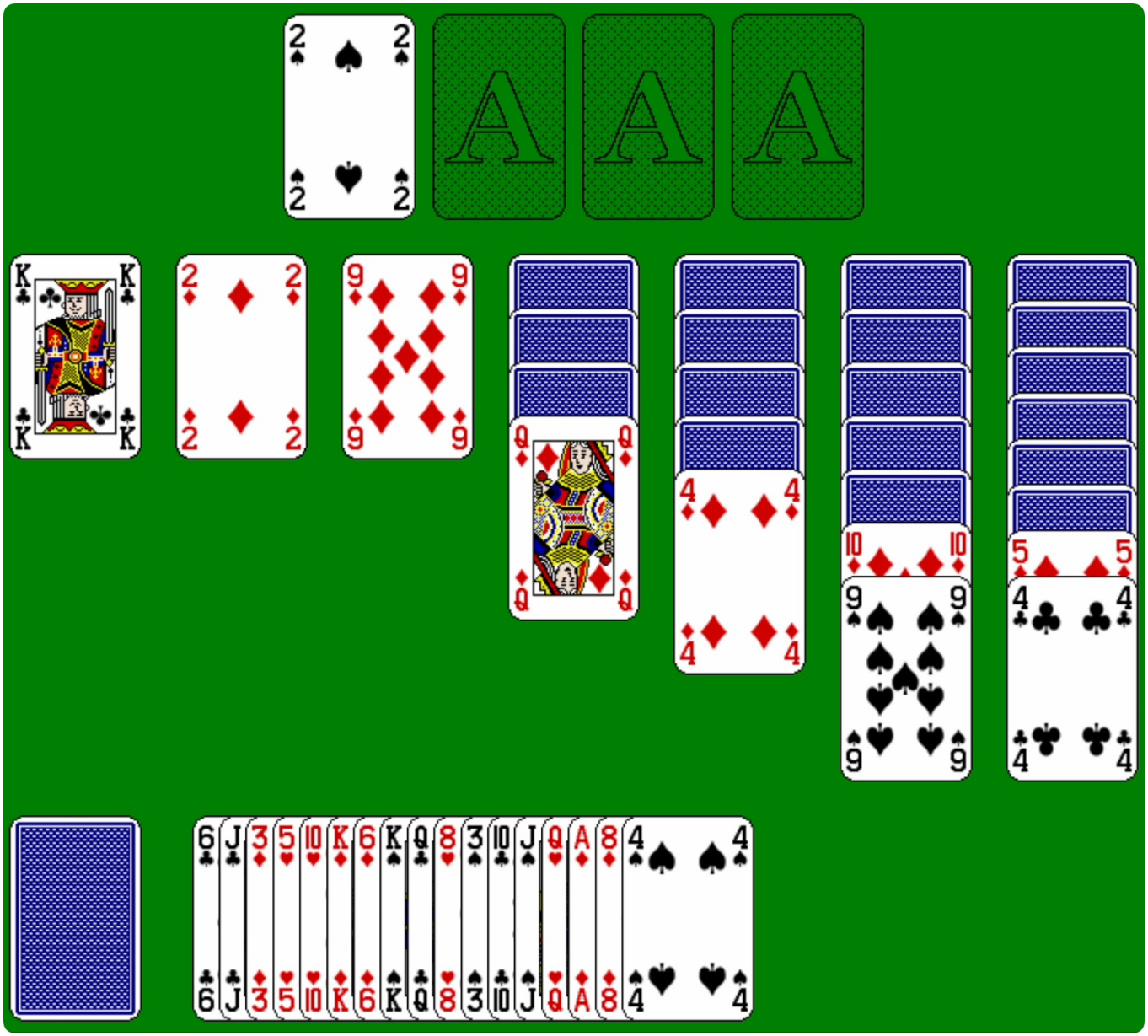

Face Up
Built Group

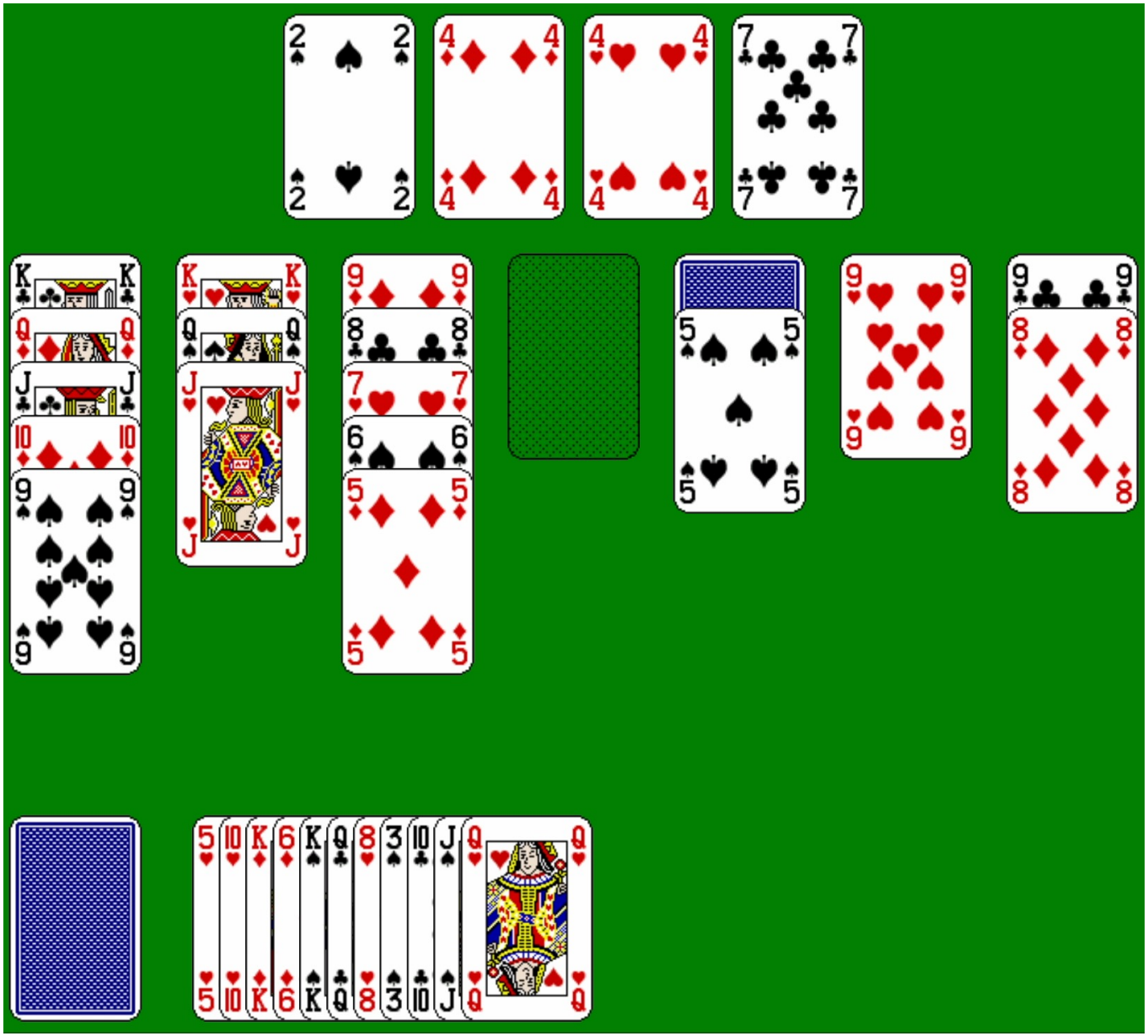


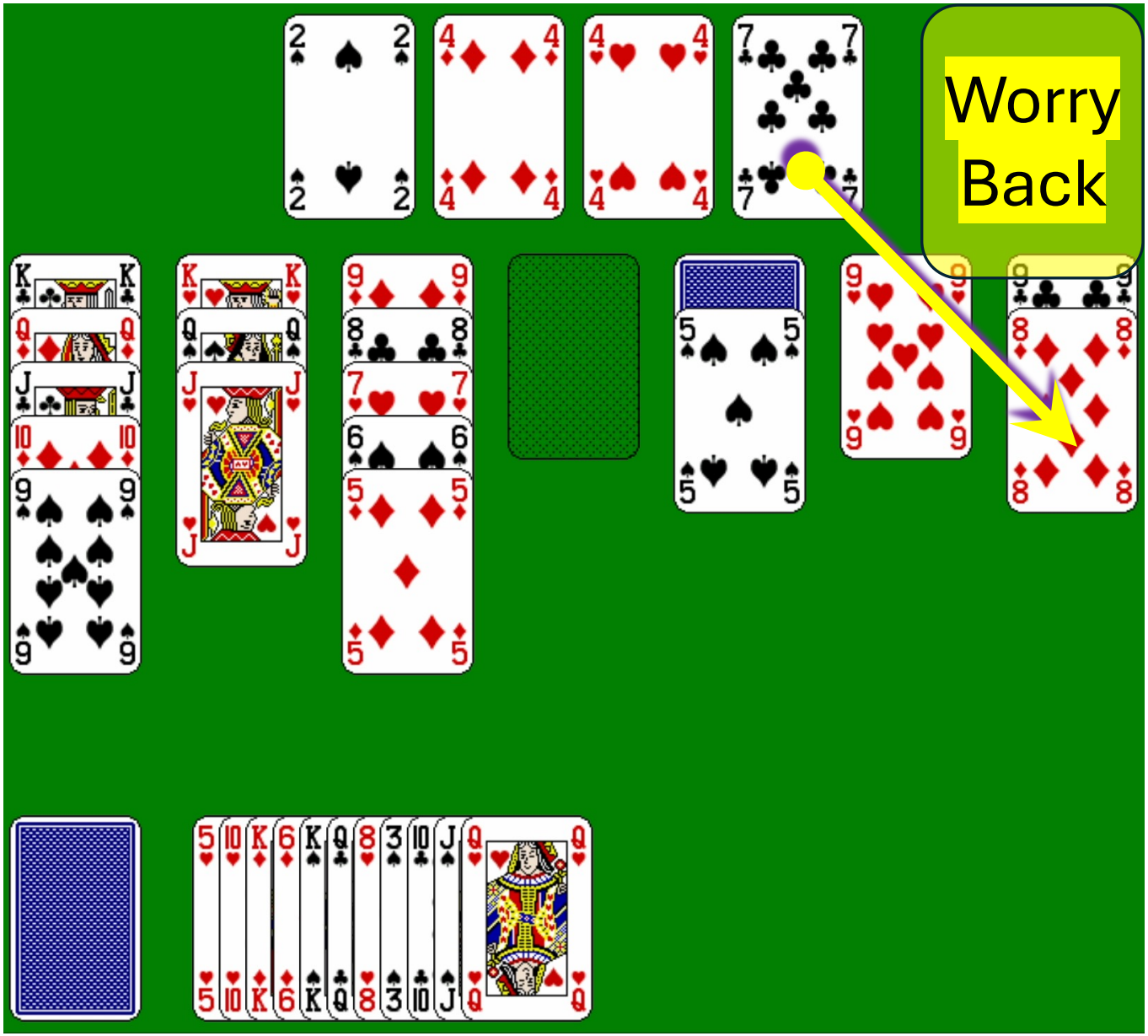


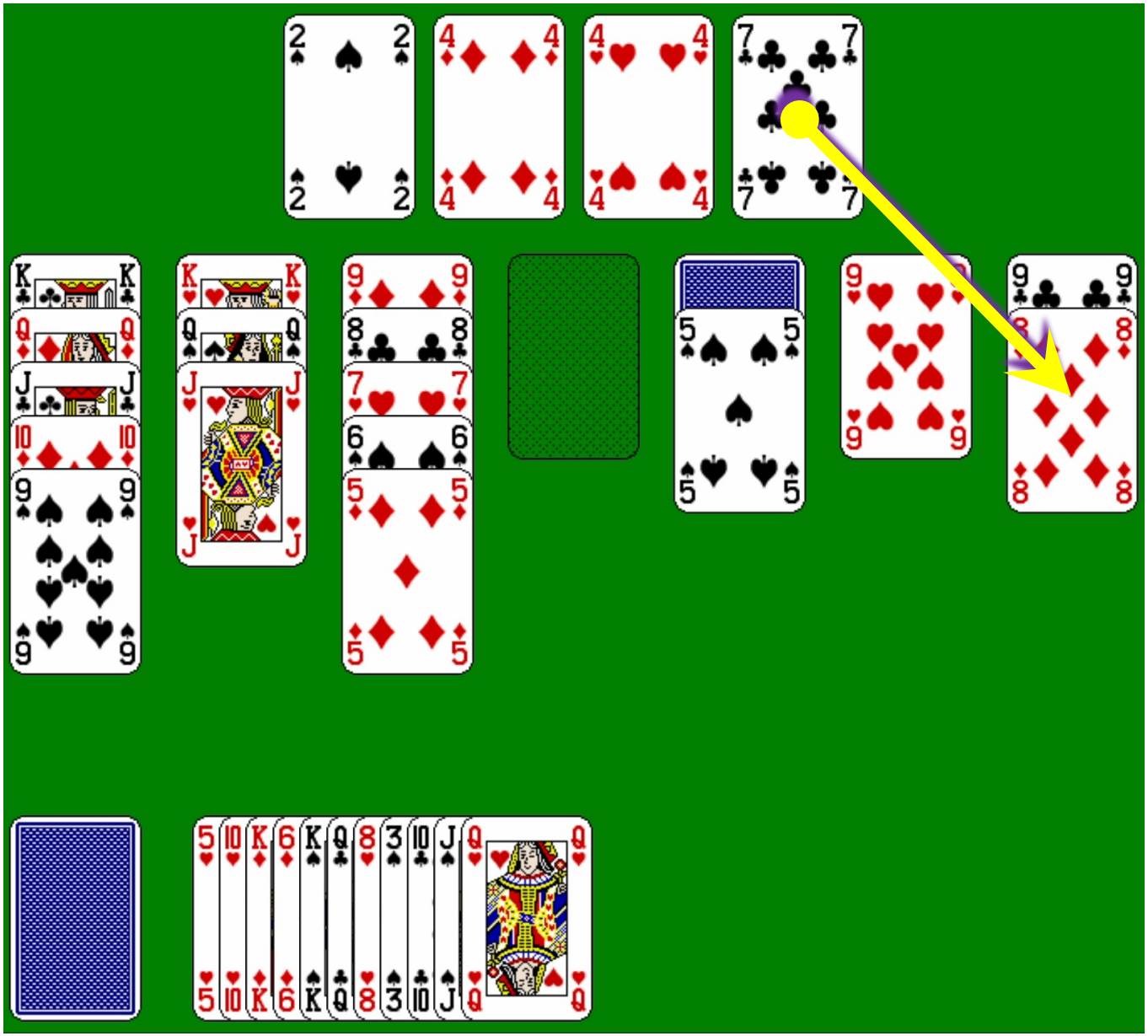


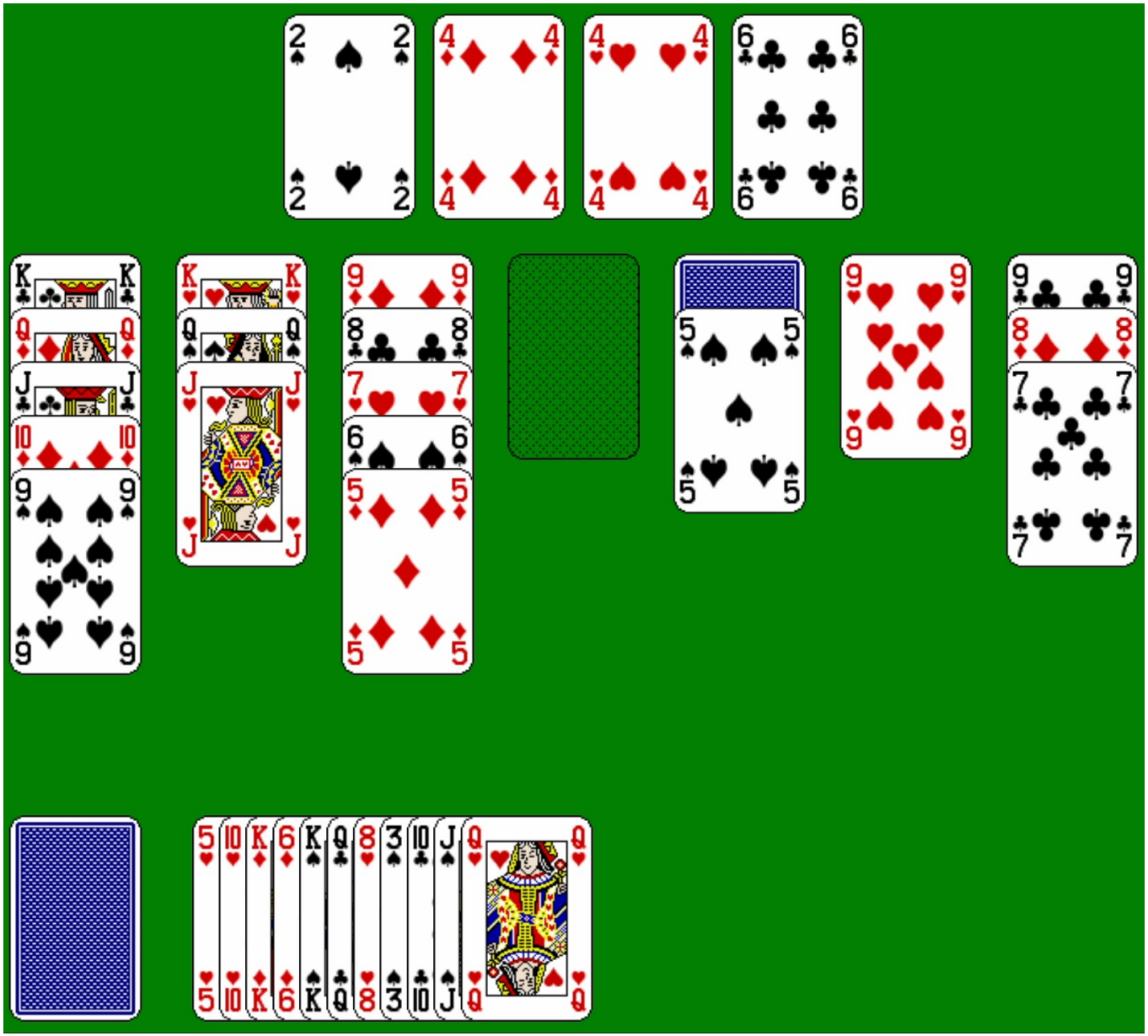


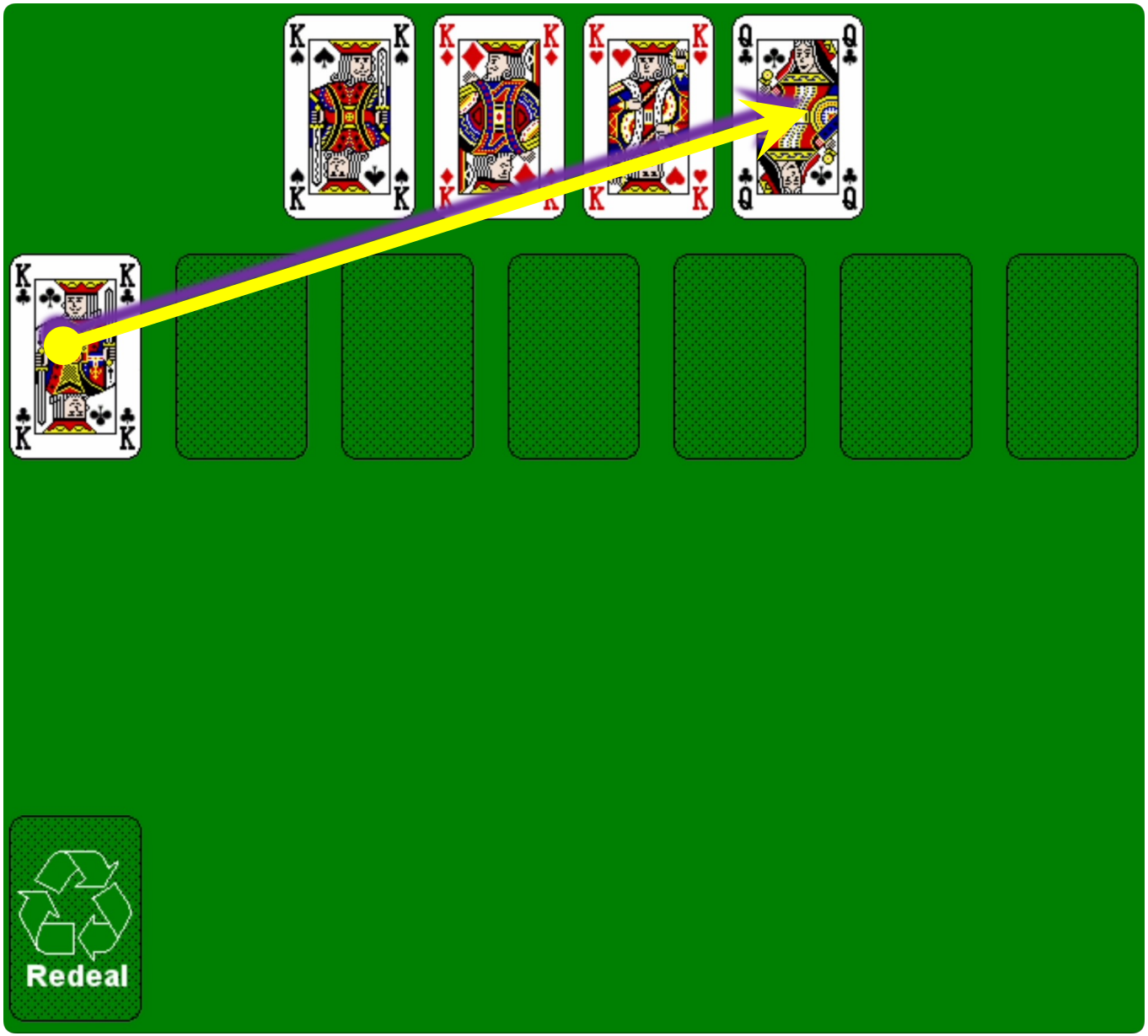


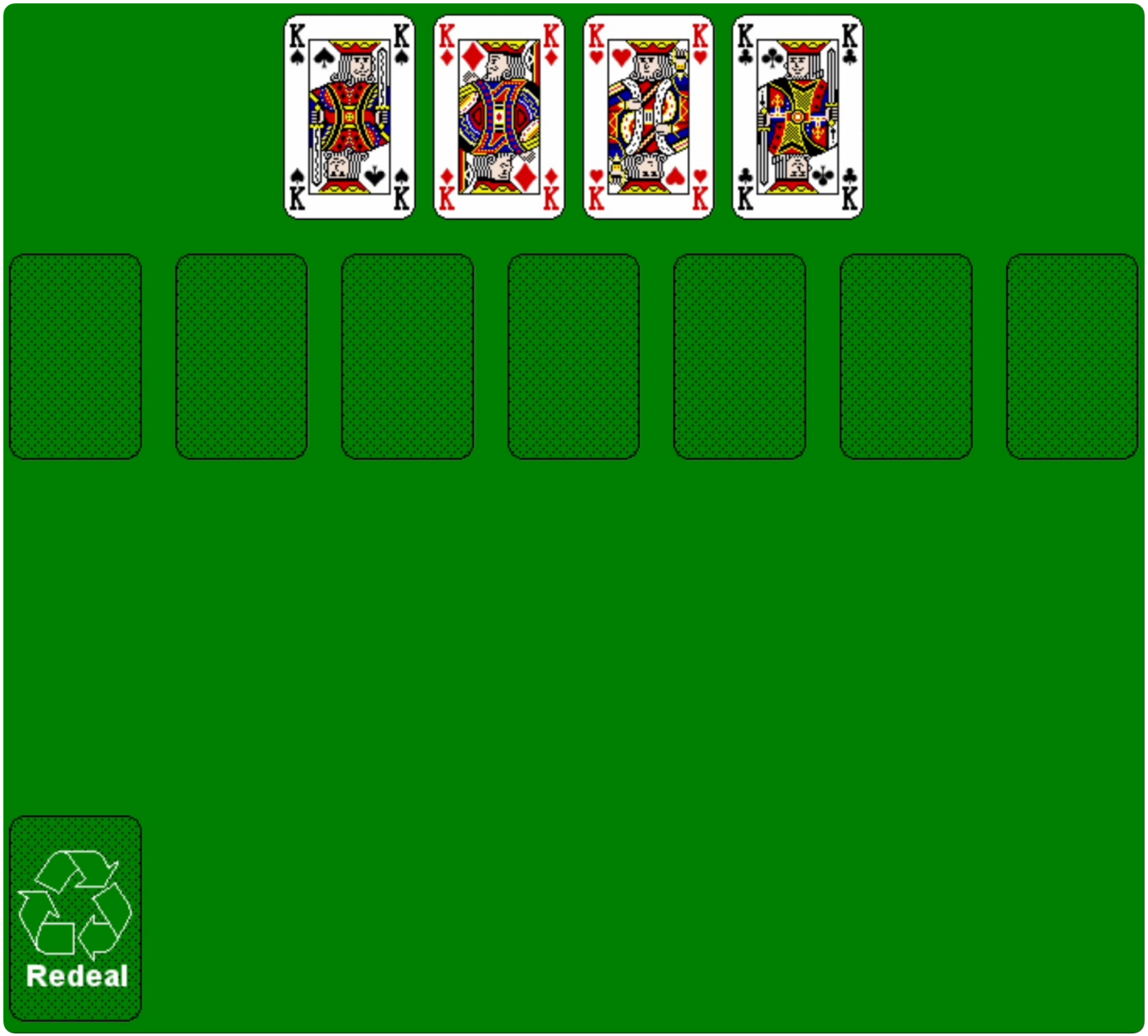



Worry
Back






Redeal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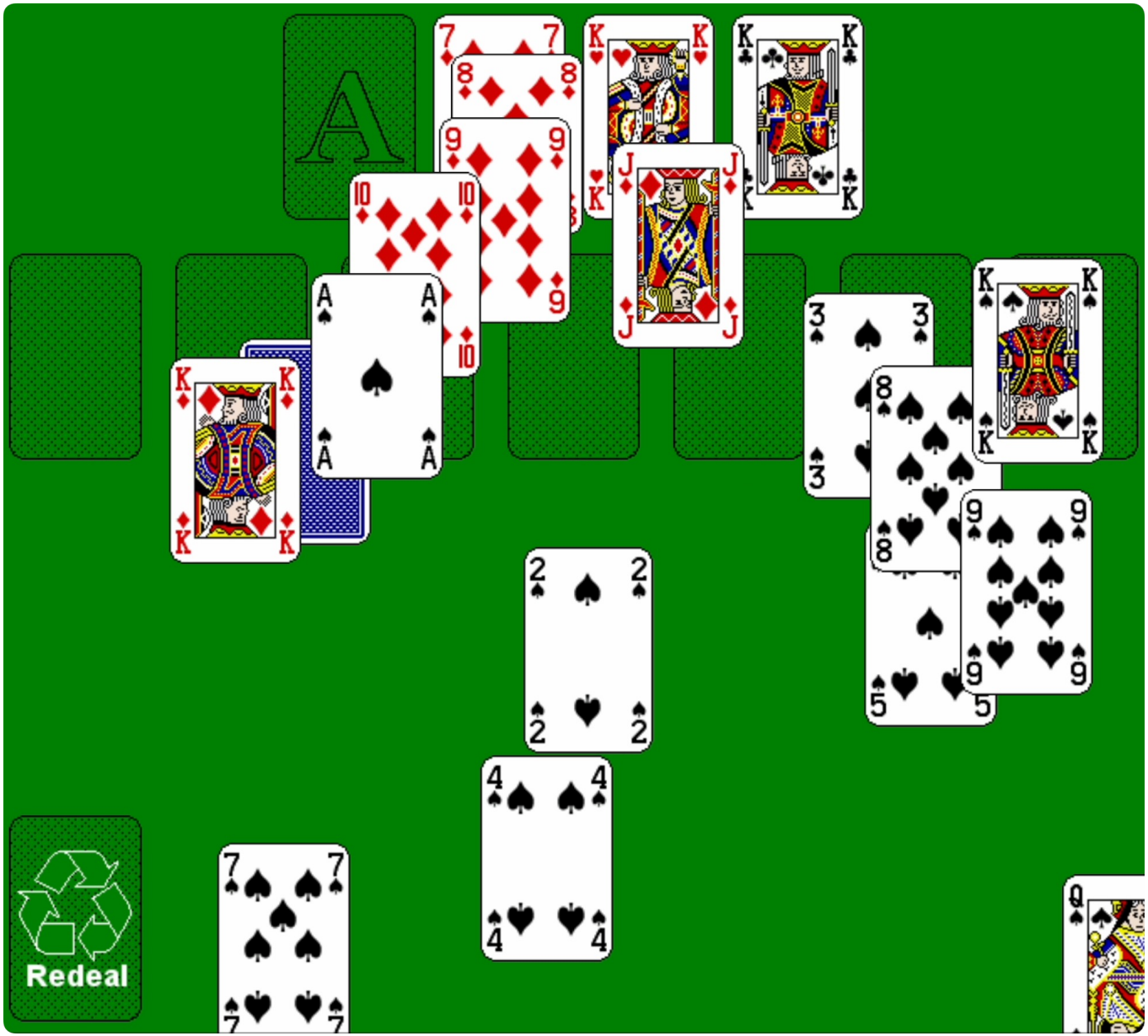
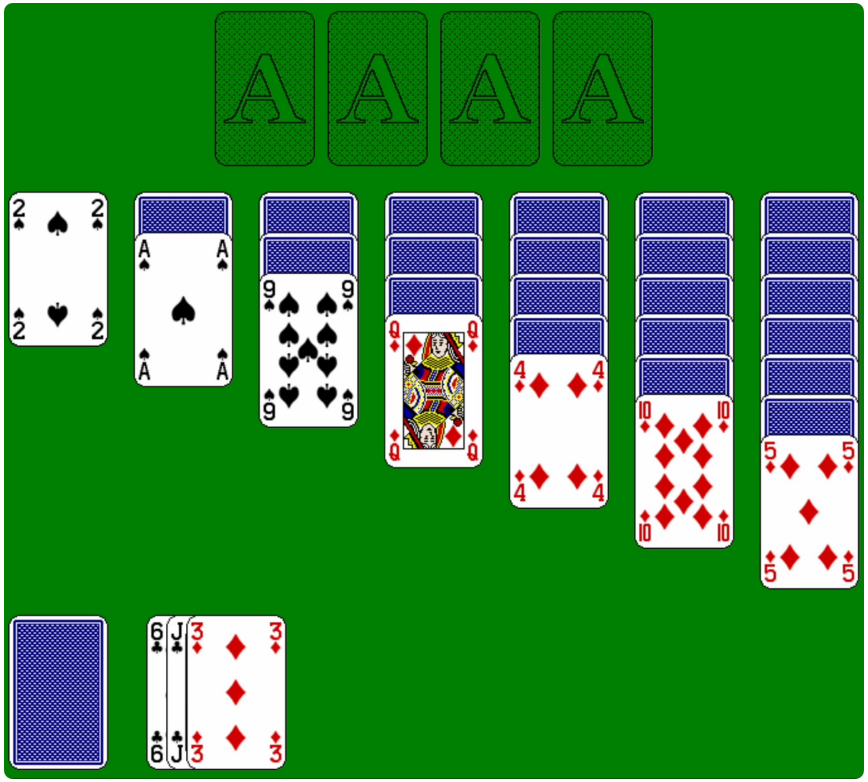

Redeal

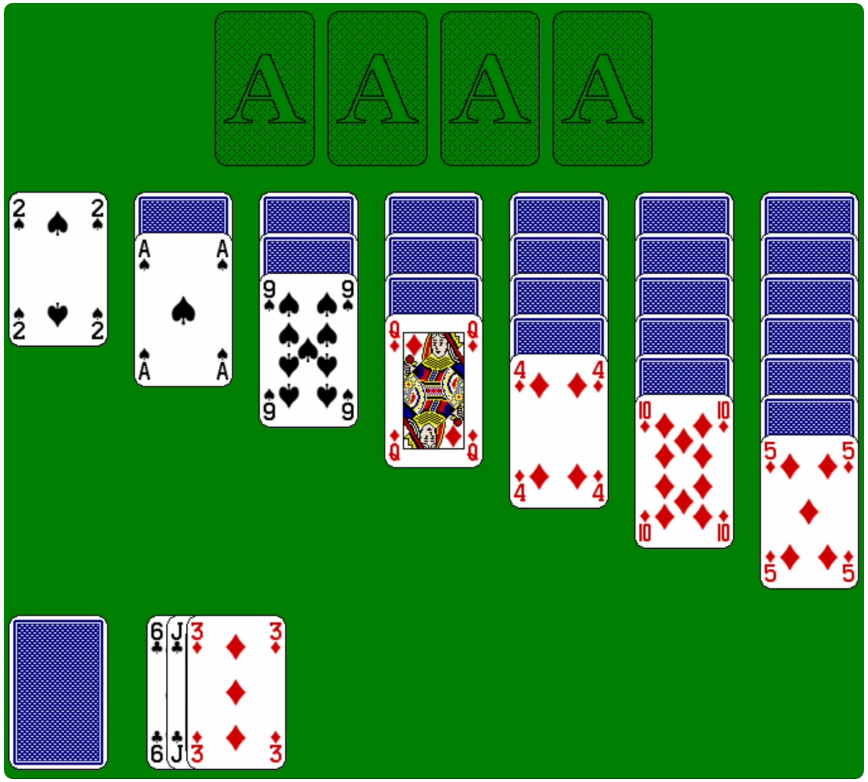
Conclusions

- The most famous single player card game is called “Klondike”
- Single player card games are called “Solitaire” or “Patience”
- There are hundreds of other Patiences

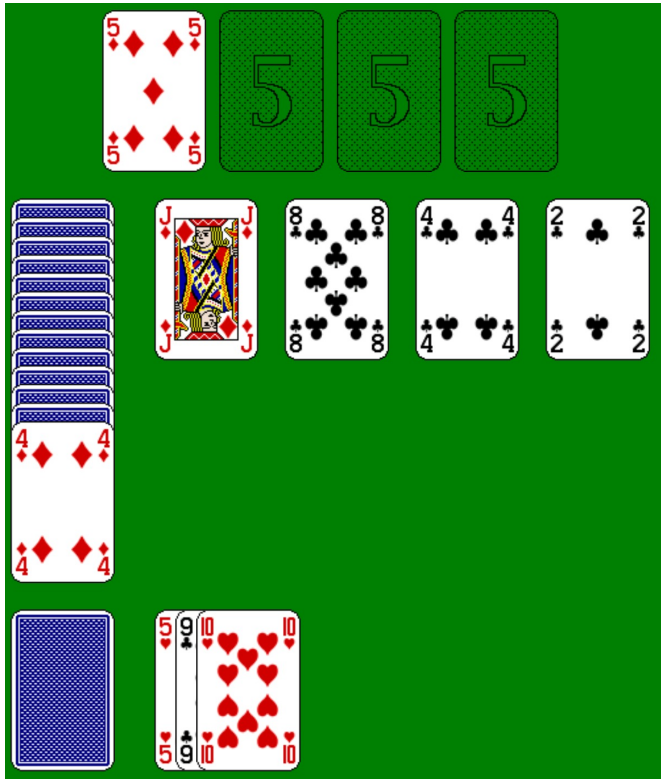
I called this game **Klondike**



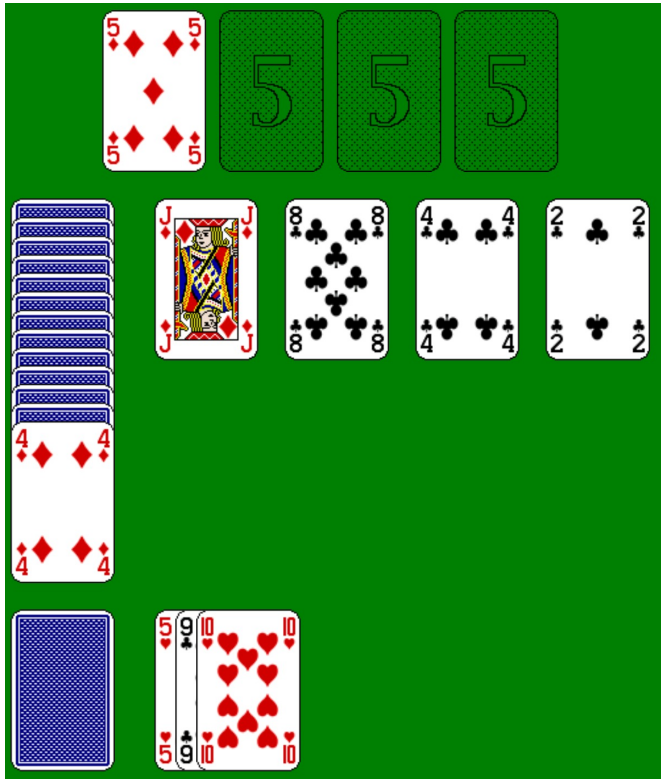
But **Klondike** is sometimes called “**Canfield**”



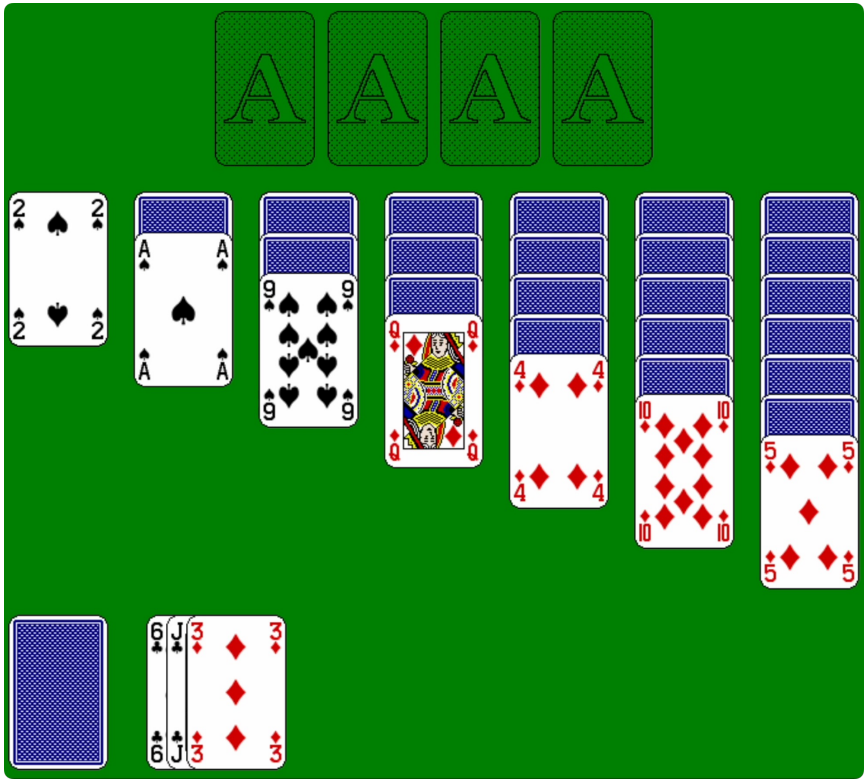
But **Canfield** is the widely used name for this game



But **Canfield** is sometimes called “**Demon**”



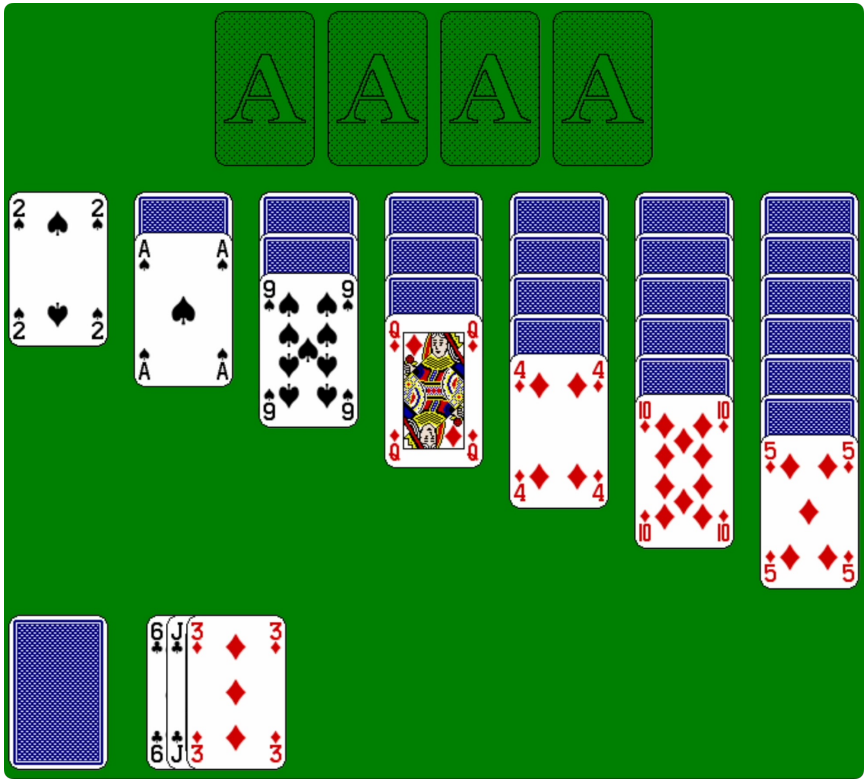
But “**Demon**” is also another name for **Klondike**



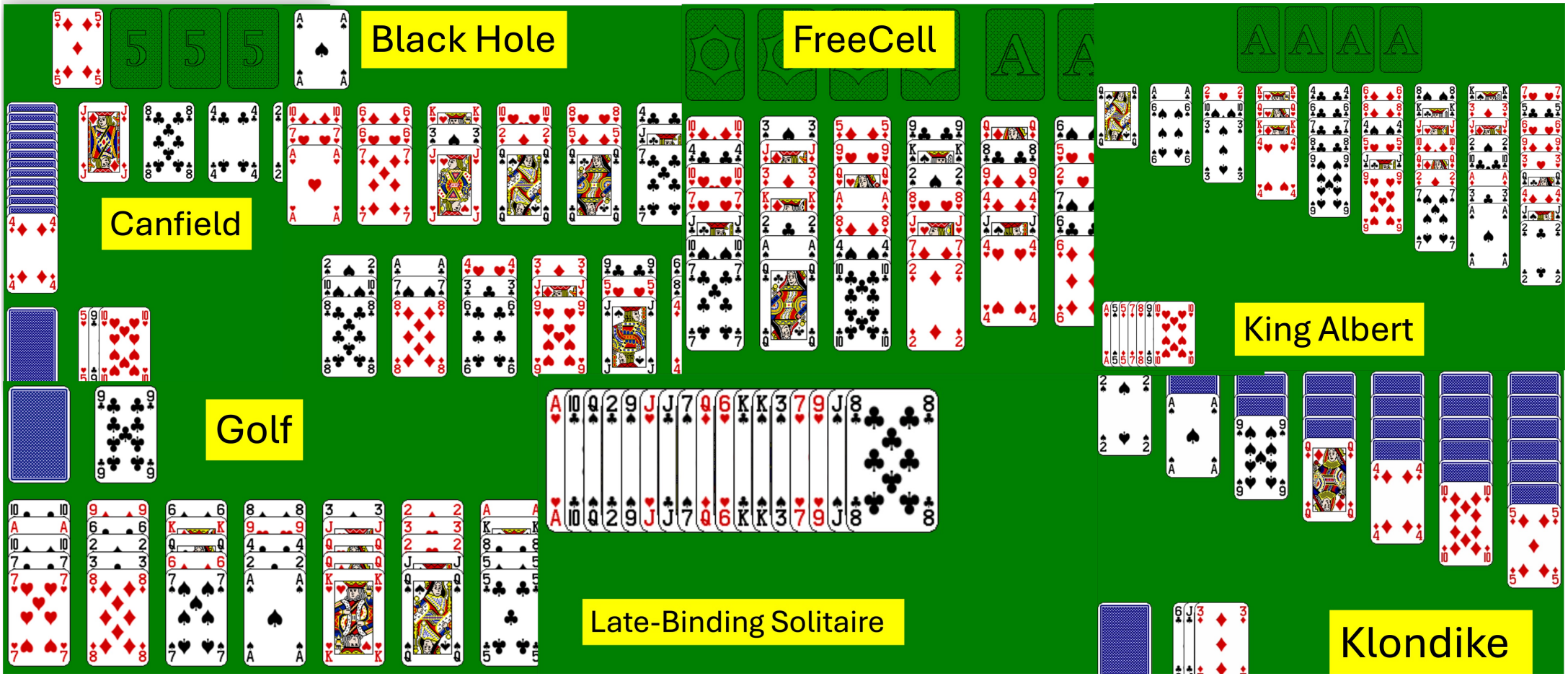
Fun Fact

- “Demon” is what my mother called Canfield
- “Demon” is what my father called Klondike

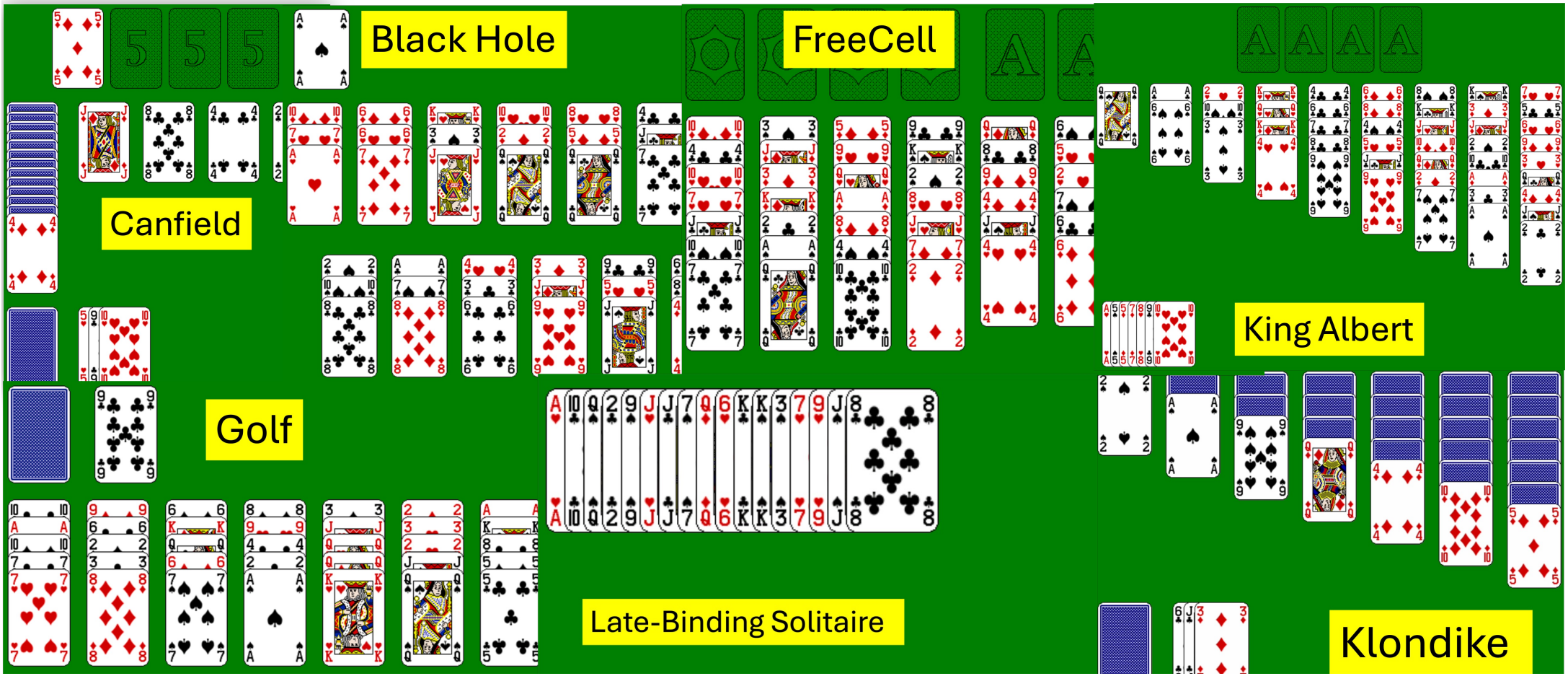
But **Klondike** is sometimes called “**Patience**”



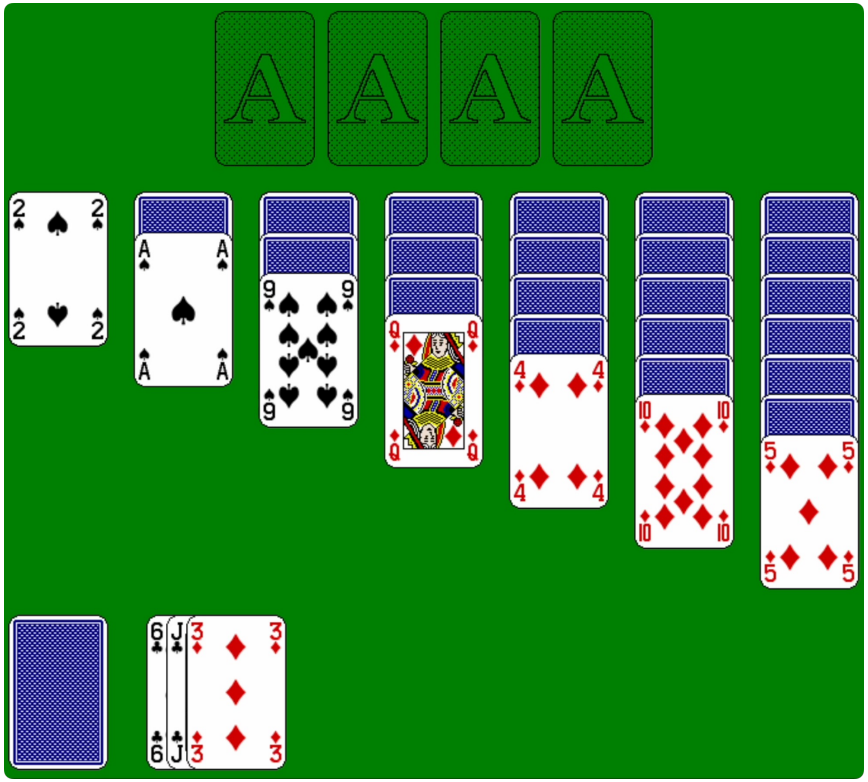
But **Patience** is also the generic name for these seven very different games and many more



But **Patience** is sometimes called “**Solitaire**”



But “**Solitaire**” is another name for **Klondike**



To keep things straight …

- I use “**Patience”** and “**Solitaire”** as synonyms for “single player card games”
  - Patience is the traditional word in British English
  - Solitaire is the traditional word in US English
- I use the generally accepted name of a game, e.g. “**Klondike”**
  - The internet has usually settled on a standard name for each game
  - But detailed rules vary even for the same game

People care about Winnability of Patience

- It’s one area where actual people care about random instances!

- Crowdsourcing was used to solve FreeCell
- Internet FreeCell Project, 1990s
  - All 32,000 deals in Microsoft FreeCell were solved by humans
  - Except for one which is impossible

Fun fact

- Crowdsourcing was used to solve FreeCell games (1994)
- …. 10 years before the word “crowdsourcing” was invented (2006)

People care about Winnability of Patience

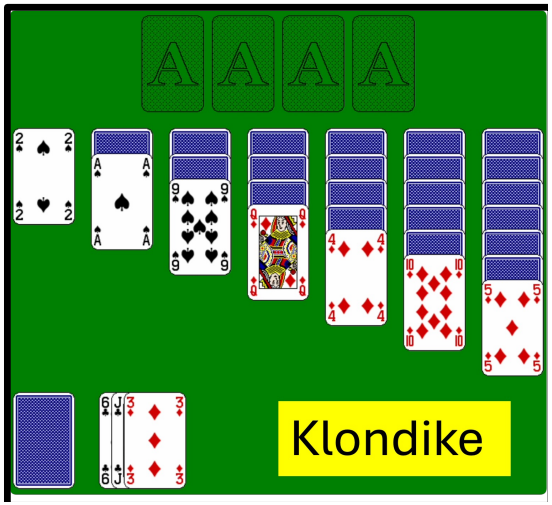
- It's one area where actual people care about random instances!
- Including distinguished scientists like Persi Diaconis

"It is one of **the embarrassments of applied mathematics** that we cannot determine the odds of winning the common game of solitaire [Klondike]."

Solitaire: Man versus Machine

Xiang Yan, Persi Diaconis, Paat Rusmevichientong, Benjamin V Roy

Advances in Neural Information Processing Systems, 2005



Winnability of Klondike is **81.945 ± 0.084%**

The 95% confidence interval is [81.861%,82.029%]

>100 times better than Yan et al's estimate

30 times better than previous best estimate: Birrell, 2017

"It is one of **the embarrassments of applied mathematics** that we cannot determine the odds of winning the common game of solitaire [Klondike]."

Solitaire: Man versus Machine

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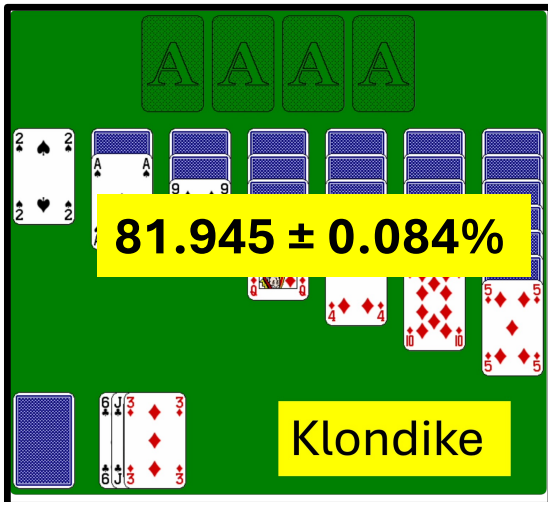
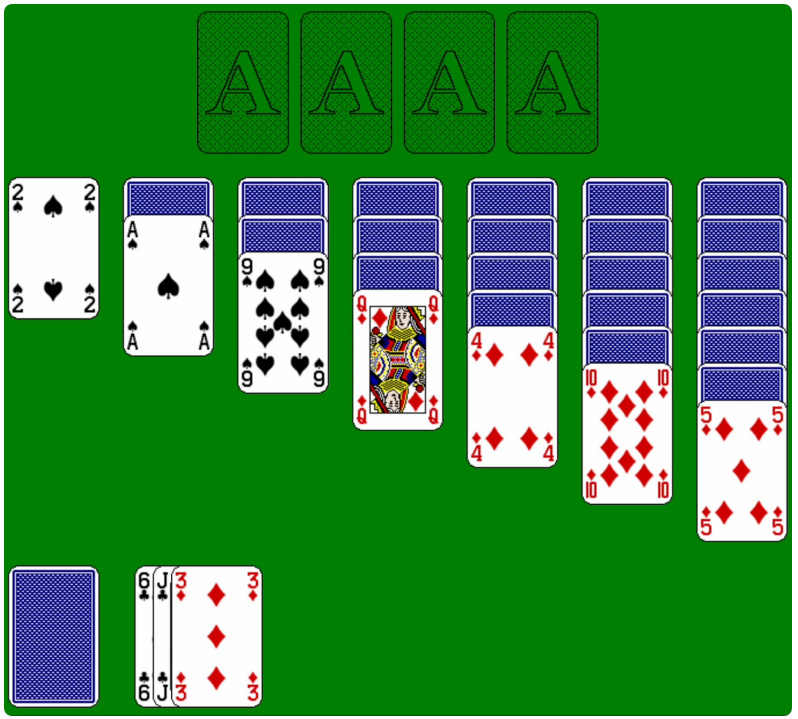


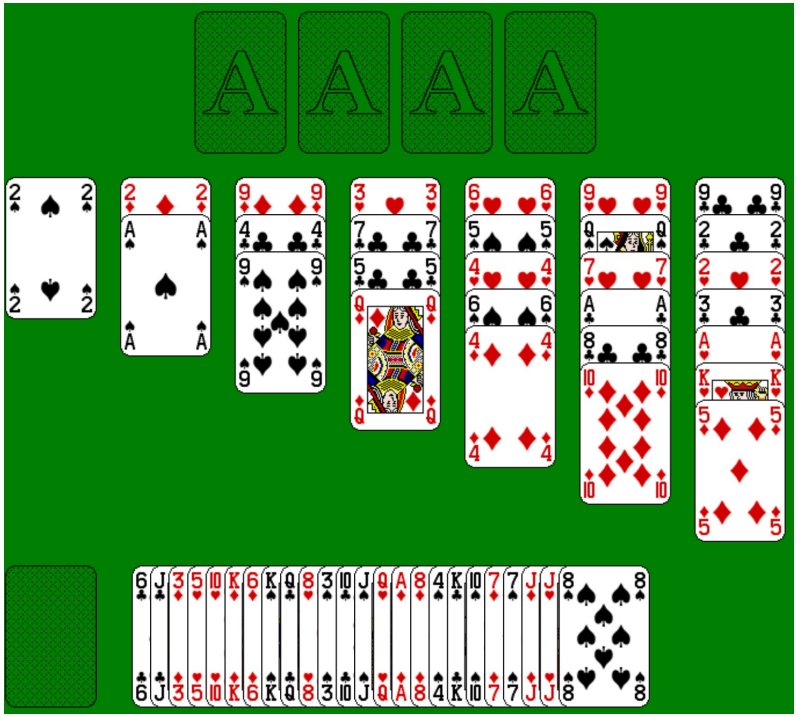
What do we mean by “Winnability”?

- Just what you would expect …
- The probability of winning a random layout with perfect play
- With **one extra assumption**



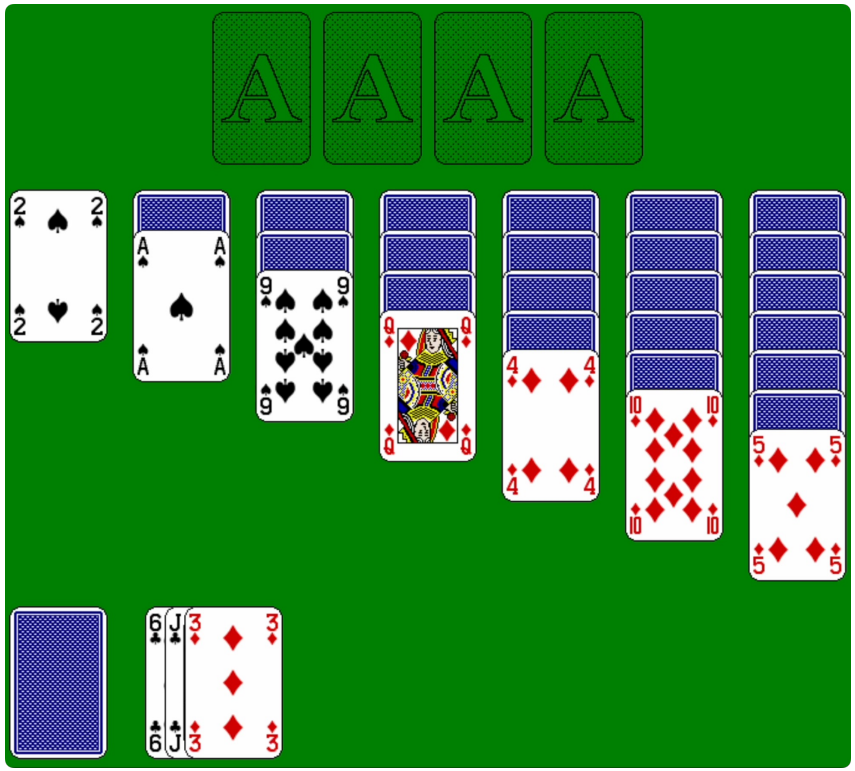
What do we mean by “Winnability”?

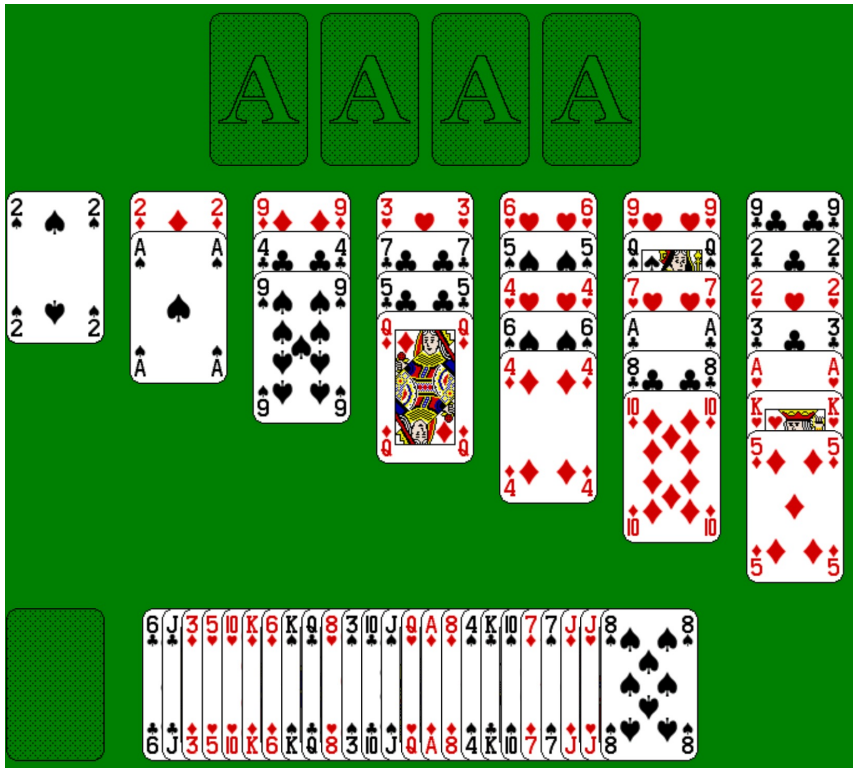
- Just what you would expect …
- The probability of winning a random layout with perfect play
- With one extra assumption
- We **know** where all the cards are at the start
- Including the hidden cards
- This is called “**Thoughtful**”
  - With real cards, peek underneath
  - On computer, use unlimited undos



A Question to Think About

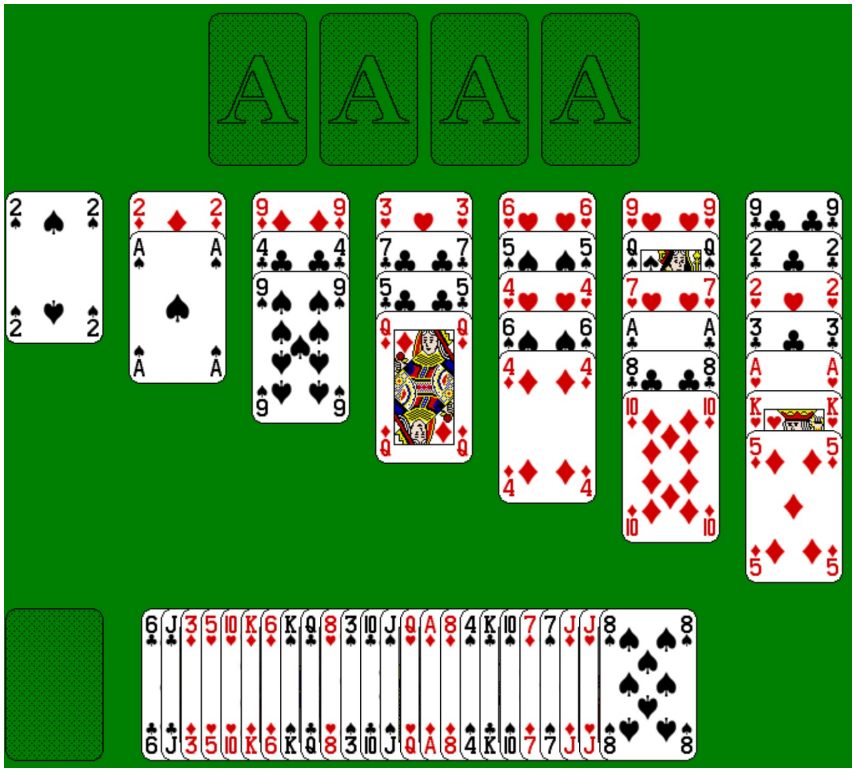
- Thoughtful Solitaire is almost *but not quite* the same as playing with all cards face up …. why? *(Hint: It's visible on this slide)*





Fun Fact

- The idea for “Thoughtful Solitaire” came from a president of the American Mathematical Society
- Specifically Irving Kaplansky



Conclusions

- The most famous single player card game is called “Klondike”
- Single player card games are called “Solitaire” or “Patience”
- There are hundreds of other Patiences
- We built a solver “Solvitaire” which can solve many many of these
- Solvitaire got world’s best results on dozens of Patience games

Our program is called **“Solvitaire”**

- Solvitaire is one of the most felicitous names ever
- I have named a few things whose names have stuck
- See “The Petrie Multiplier”
- But when Charlie suggested “Solvitaire” it was game over…

Fun Fact!

- Charlie told me the above slide was a lie!
- I actually said I didn’t like the name
- But was soon won over

Bonus Fun Fact!

- My best ever name suggestion was “Large Neighbourhood Search”
- Fortunately, Paul Shaw told me I do remember that one right!

Paul Shaw

FOLLOW

IBM
Verified email at fr.ibm.com
Constraint programming optimization AI packing routing

TITLE	CITED BY	YEAR
Using constraint programming and local search methods to solve vehicle routing problems P Shaw International conference on principles and practice of constraint …	2232	1998

Solvitaire Winnability Estimates

- We use Monte Carlo methods to estimate winnability
- E.g. run 1,000,000 random games
- Look at number of won/lost/indeterminate
- Compute a 95% confidence interval on the true winnability
- Monte Carlo methods were invented by Stanislaw Ulam
  - (Don’t forget this, it’s foreshadowing)

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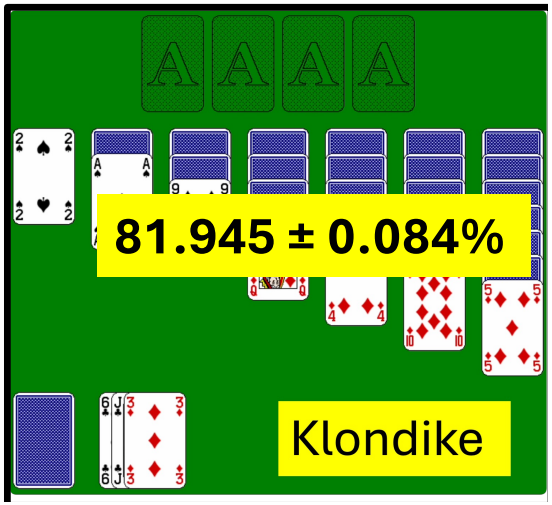
Solvitaire Winnability Estimates

Winnability of **Klondike** is **81.945 ± 0.084%**

30 times better than previous best estimate: Birrell, 2017



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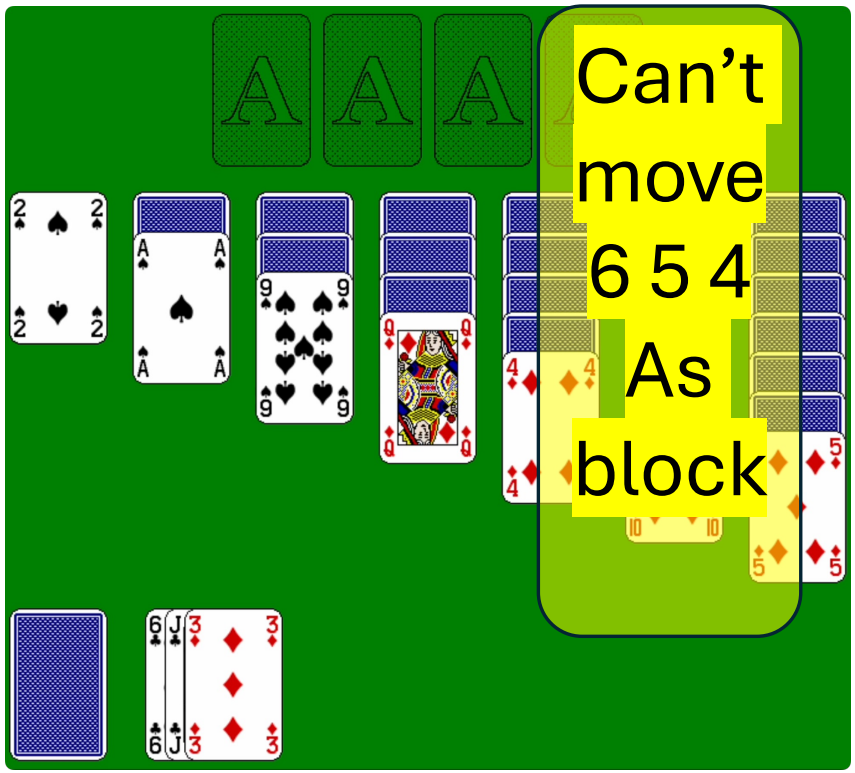
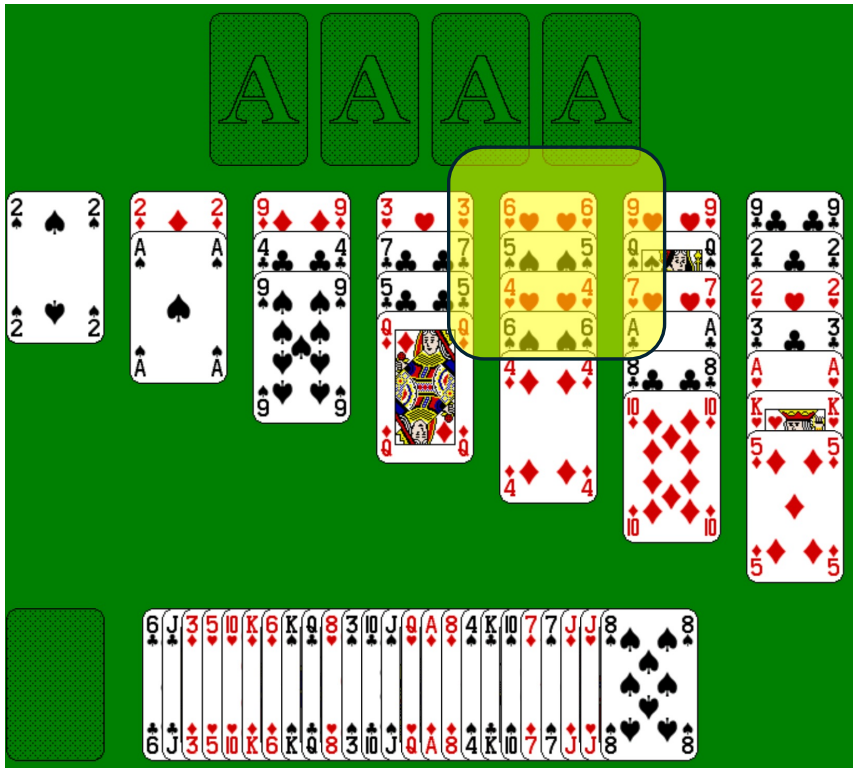
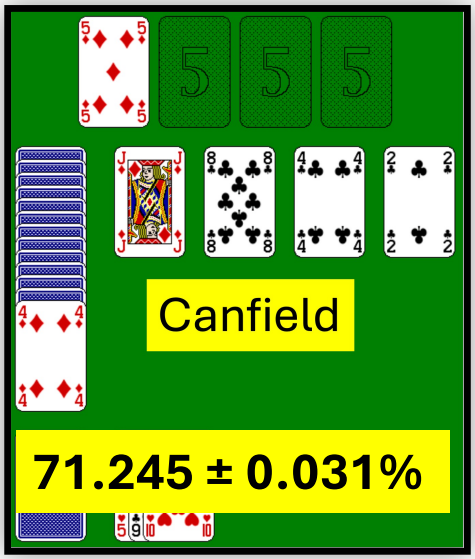




Solvitaire Winnability Estimates



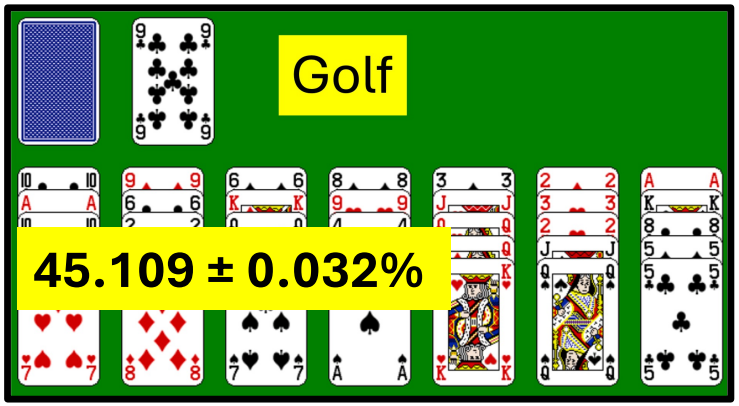
Winnability of **Canfield** is **71.245 ± 0.031%**

Also more than 30 times better than (Wolter, 2013)

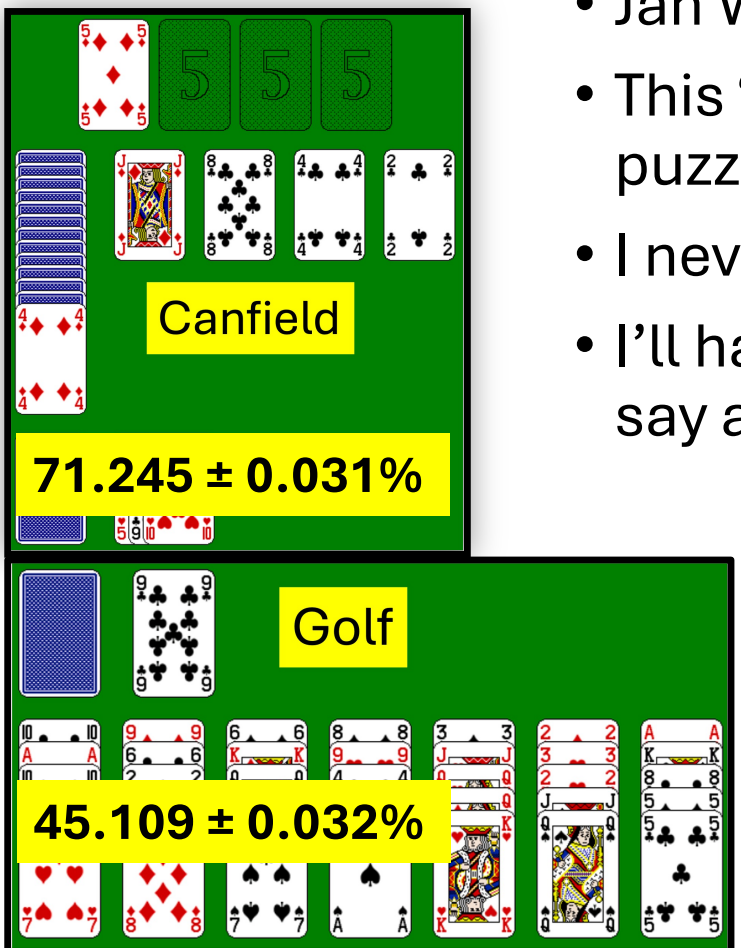
Solvitaire Winnability Estimates

Winnability of **Golf** is **45.109 ± 0.032%**

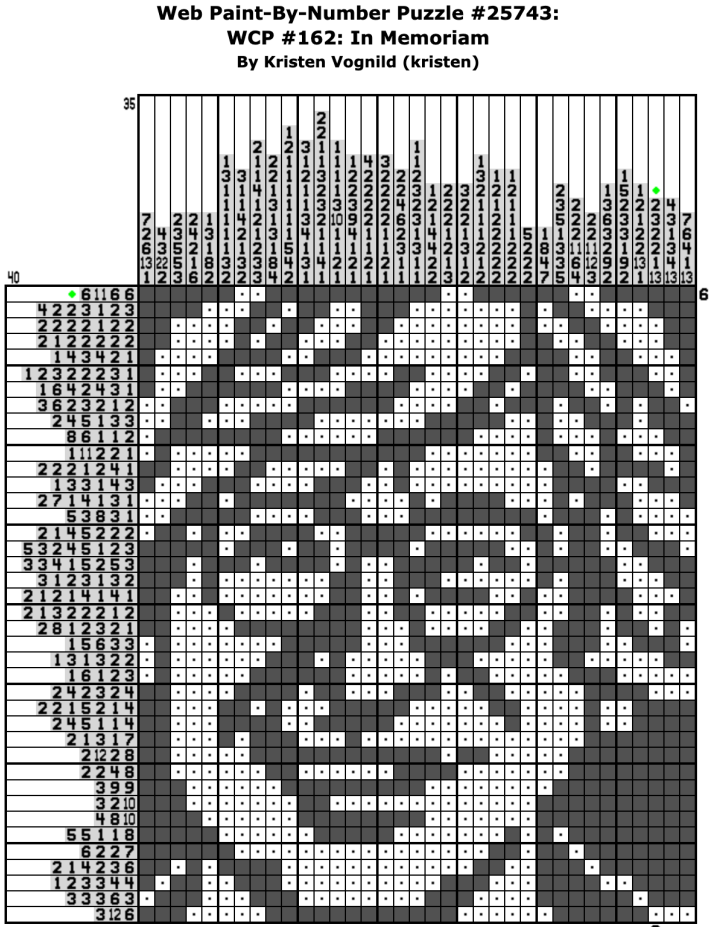
“Only” 10 times better than (Wolter, 2013)



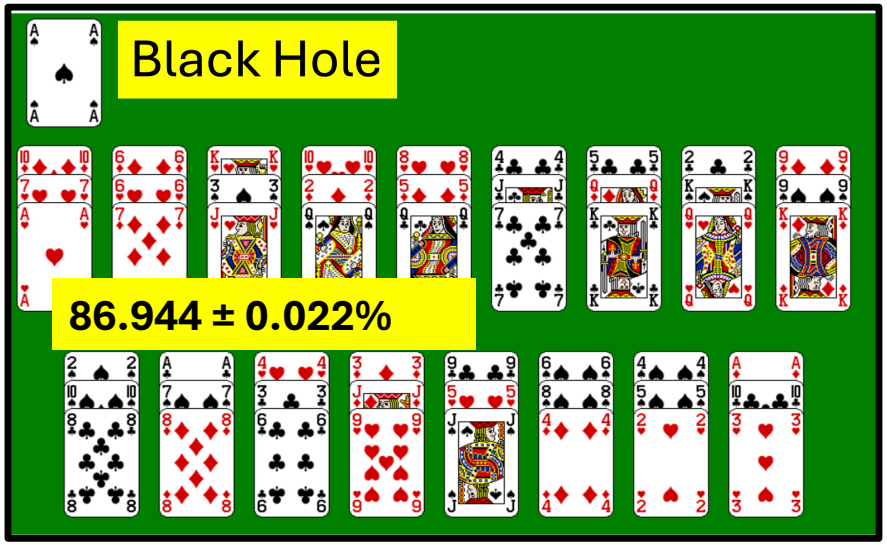
Not so Fun Fact



- Jan Wolter died 1/1/2015
- This “paint by numbers” puzzle was a tribute to him
- I never met him
- I’ll have more nice things to say about Jan in a bit



Solvitaire Winnability Estimates



- For Black Hole, we get **86.944 ± 0.022%**
- Only 2 times better than
- 86.986 ± 0.053%

(Masten, 2022)

Solvitaire Winnability Estimates

- For King Albert, we get **68.542 ± 0.092%**
- Which is 90 times better than
- 71.189 ± 8.678% (Roscoe 2016)

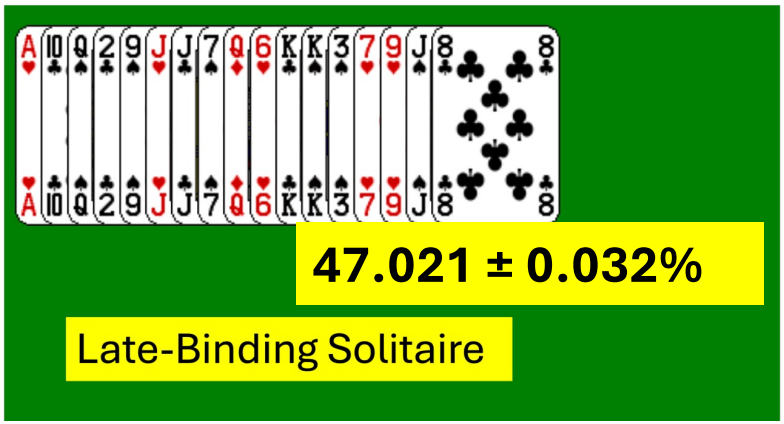
Fun Fact

- King Albert was my mother’s favourite patience
- She once swapped open-air cockpits in mid air during WW2

Photo: LA(PHOT) Abbie Herron/MOD

Solvitaire Winnability Estimates

- For Late-Binding Solitaire, we get **47.021 ± 0.032%**
- Which is 90 times better than
- 45.418 ± 3.081% (Ross & Knuth 1989)



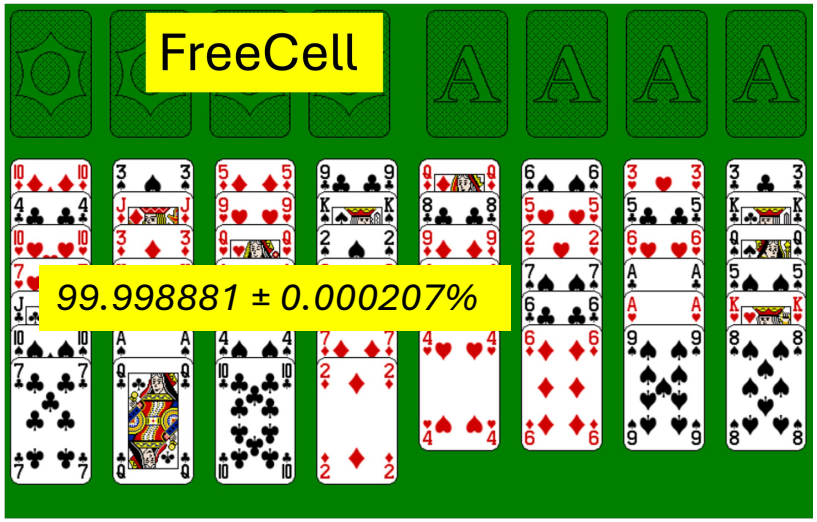
Fun Fact

- Donald Knuth doesn’t have an email address
- Which made me very surprised to get an email from him asking what Late-Binding Solitaire was
- (it’s a variant of “Accordion”)

Solvitaire Winnability Estimates

- For Freecell we got

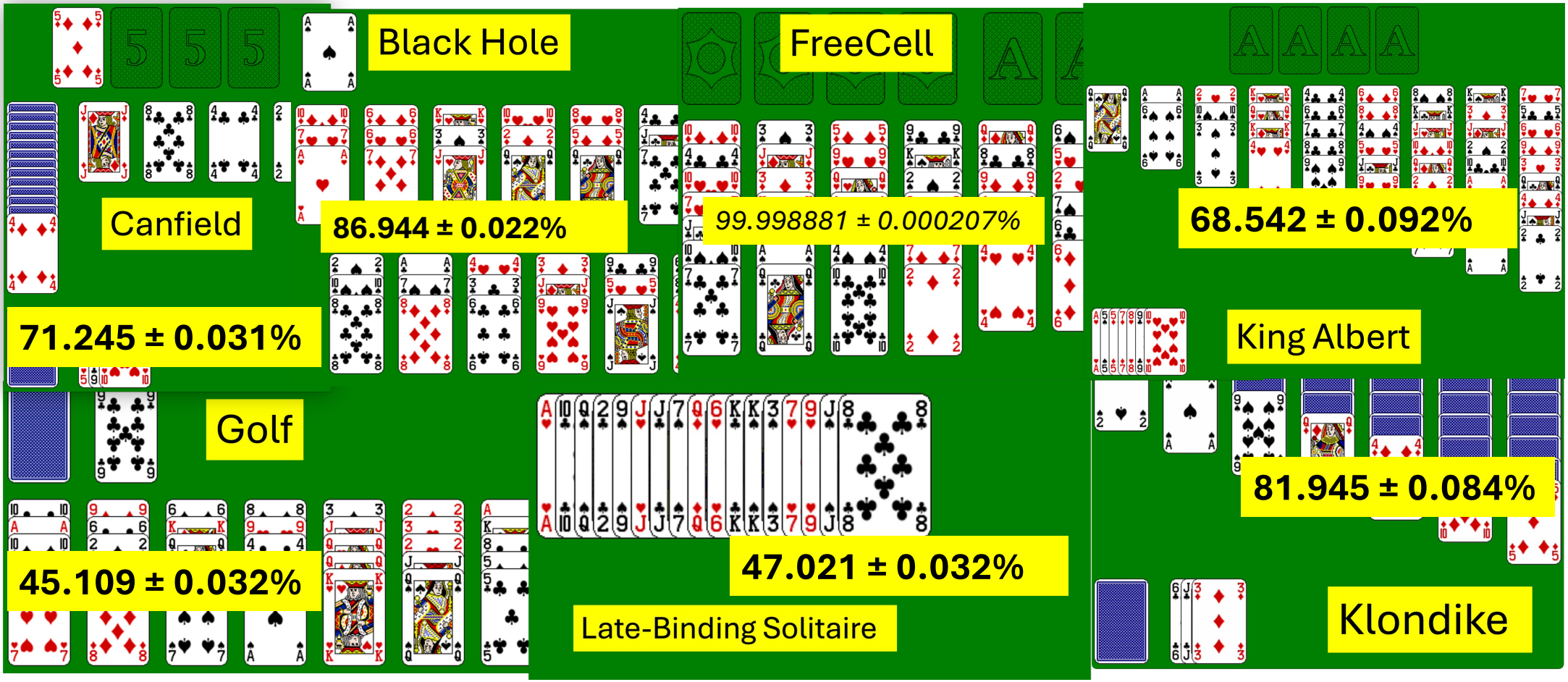
99.998881 ± 0.000207%



- Which is 25 times *worse* than

99.998812 ± 0.000008% (Fish 2018)

Solvitaire Winnability Estimates



22 more games nobody had tried before

Game	Confidence Interval Percentage Range
Alpha Star	47.794% ± 0.032%
American Canister	5.606% ± 0.015%
Beleaguered Castle	68.170% ± 0.099%
British Canister	0.000129% ± 0.000008%
Canfield (Whole Pile Moves) [*Th.*]	67.562% ± 0.034%
Carpet [*Th.*]†	87.558% ± 0.021%
– " – (Pre-founded Aces) †[*Th.*]	95.186% ± 0.014%
Delta Star	34.413% ± 0.030%
East Haven [*Th.*]	82.844% ± 0.100%
Fan	48.776% ± 0.099%
Fortune's Favor [*Th.*]	99.9999879% ± 0.0000022%
Mrs Mop	97.992% ± 0.079%
Northwest Territory [*Th.*]	68.369% ± 0.094%
Raglan	81.226% ± 0.085%
Siegecraft	99.136% ± 0.020%
Somerset	53.725% ± 0.097%
Spanish Patience	99.863% ± 0.003%
Spiderette [*Th.*]	99.620% ± 0.018%
Streets and Alleys	51.187% ± 0.186%
Stronghold	97.379% ± 0.042%
Thirty	67.454% ± 0.030%
Will O' The Wisp [*Th.*]	99.9240% ± 0.0027%

22 more games nobody had tried before

- Many books have given estimates of winnability in patience games
- Some have been **wildly inaccurate**
  - (British) Canister was described by Parlett (1980) as “odds in favour”
- Some have been **wildly accurate**
  - Cavendish (1890) said that Fan “with careful play, is slightly against the player”

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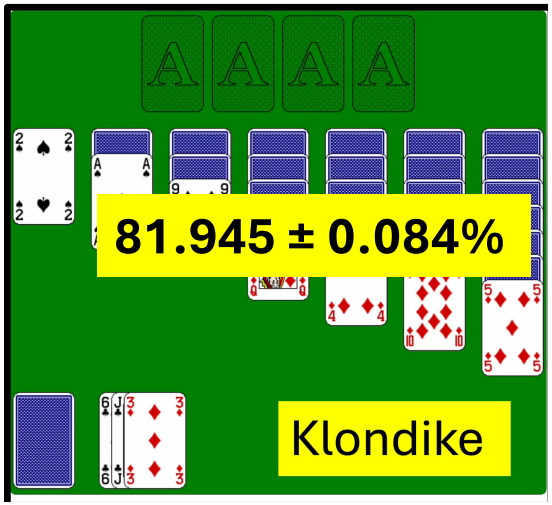
- Single player card games are called “Solitaire” or “Patience”
- There are hundreds of other Patiences
- We built a solver “Solvitaire” which can solve many many of these
- Solvitaire got world’s best results on dozens of Patience games
- Uses just a basic JSON description of rules

Solvitaire is not hardwired for specific games

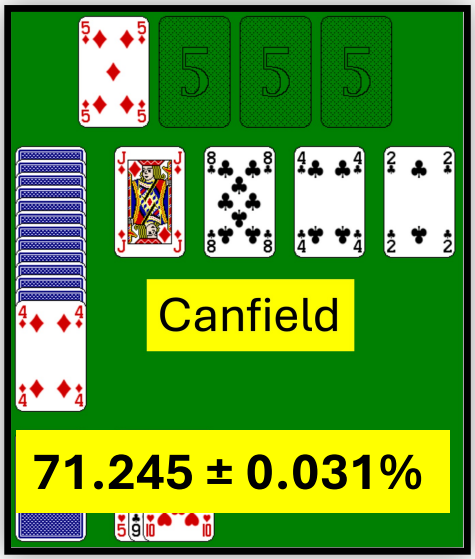
- Solvitaire is very general
- Compare to wide range of solvers previously written
  - Obviously very specialized to a particular game
- Following slides shows complete rule specification per game
- … with nothing hardwired internally for these games beyond the generic rules.

JSON Rules for Klondike

```
"tableau piles": {
    "count": 7,
    "build policy": "red-black",
    "spaces policy": "kings",
    "move built group": "partial-if-card-above-buildable",
    "diagonal deal": true,
    "face up cards": "top"},
"foundations": {"removable": true},
"stock": {
    "size": 24,
    "deal count": 3,
    "redeal": true}
```

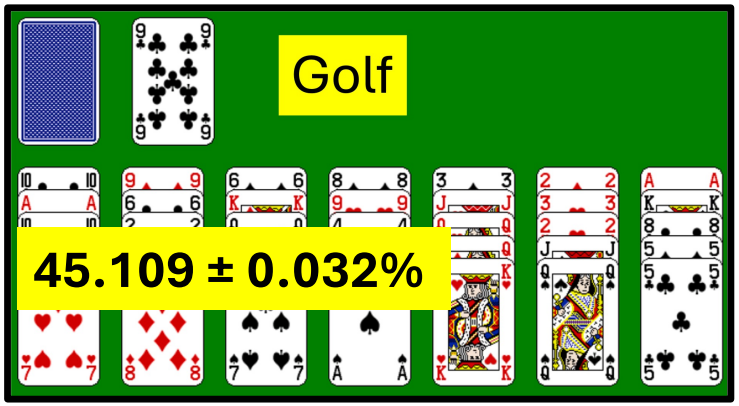


JSON Rules for Canfield



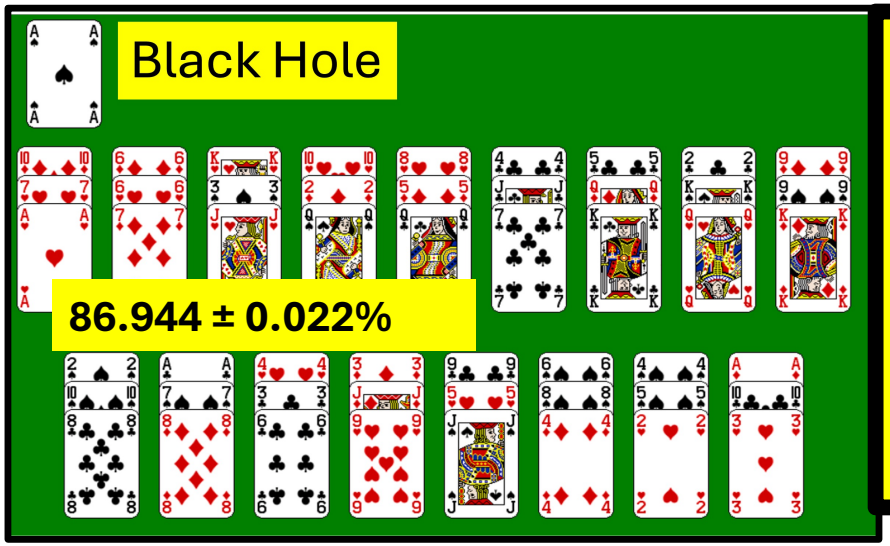

```
"tableau piles": {
    "count": 4,
    "build policy": "red-black",
    "move built group": "whole-pile",
    "spaces policy": "auto-reserve-then-waste"},
  "foundations": {
    "initial cards": "one",
    "base card": "random"},
"stock": {
     "size": 34,
    "deal count": 3,
    "redeal": true},
"reserve": {
    "size": 13,
    "stacked": true}
```

JSON Rules for Golf




```
"tableau piles": {
  "count": 7,
  "build policy": "no-build"},
"foundations": {
  "present": false},
"stock": {
  "size": 16,
  "deal type": "hole"},
"hole": {
  "present": true,
  "base card": "random",
  "build loops": false}
```

JSON Rules for Black Hole



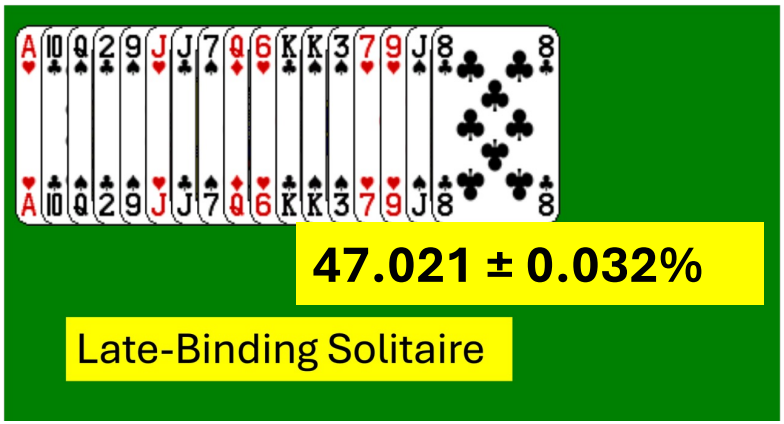

```
"tableau piles": {
  "count": 17,
  "build policy": "no-build"
},
"foundations": {
  "present": false
},
"hole": {
  "present": true
```

JSON Rules for King Albert

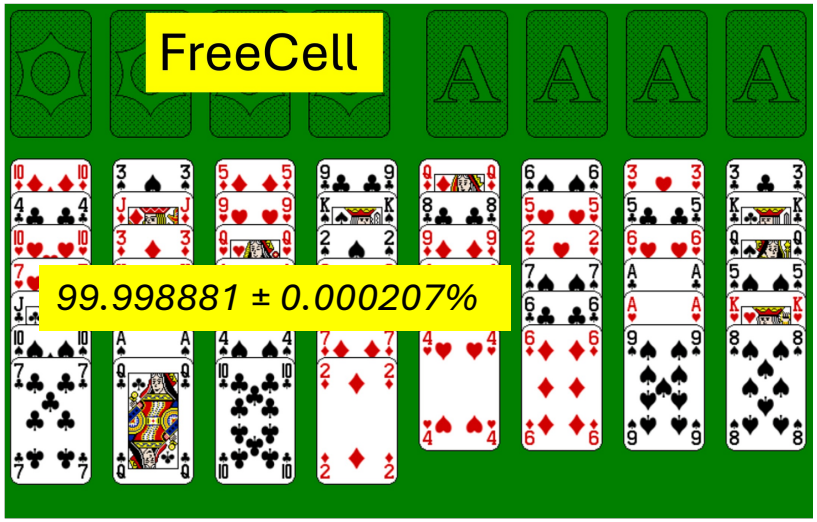
```
"tableau piles": {
  "count": 9,
  "build policy": "red-black",
  "diagonal deal": true},
"foundations": {
  "removable": true },
"reserve": {
  "size": 7 }
```

JSON Rules for Late-Binding Solitaire

```
"foundations": {
   "present": false},
 "tableau piles": {
   "count": 0},
 "accordion": {
   "size": 18,
   "moves": ["L1", "L3"],
   "build policies": ["same-suit", "same-rank"]}
```



JSON Rules for FreeCell




```
"tableau piles": {
   "build policy": "red-black"},
 "cells": 4
```

There must be a trick?!

- Default values are used which apply unless overridden
  - Does make JSON specs shorter

- Apart from that, there’s no trick
  - Every run parses the JSON rules for that game
- Once parsed the game parameters are stored internally
  - And consulted as necessary at every node in search

Conclusions

- The most famous single player card game is called “Klondike”
- Single player card games are called “Solitaire” or “Patience”
- There are hundreds of other Patiences
- We built a solver “Solvitaire” which can solve many many of these
- Solvitaire got world’s best results on dozens of Patience games
- Uses just a basic JSON description of rules
- Solvitaire is a classic example of “Good Old Fashioned AI”
  - Depth-First Search with some GOFAI add-ons
  - Transposition Tables, Symmetries, Dominances, Streamliners

GOFAI: Good Old-Fashioned AI

“GOFAI ("Good old fashioned artificial intelligence") is classical symbolic AI, as opposed to other approaches, such as neural networks, situated robotics, narrow symbolic AI or neuro-symbolic AI.”

- From Wikipedia

GOFAI vs LLMs (My cynical view)

- GOFAI
  - We can only answer a fairly limited set of questions
- LLM
  - We can answer any question you like!

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GOFAI vs LLMs (My cynical view)

- GOFAI
  - We can only answer a fairly limited set of questions
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  - We can answer any question you like!

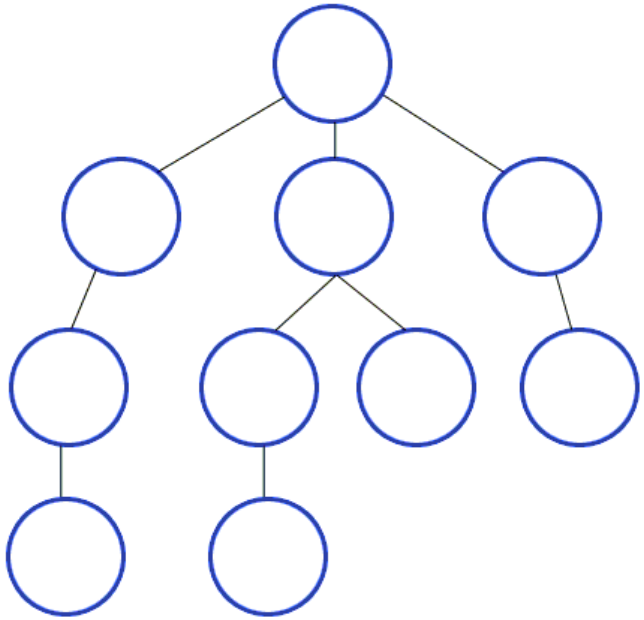
- GOFAI
  - If we can answer your question, then the answer is right
- LLM
  - What is a right answer?

Solvitaire and how it uses GOFAI

- The core of Solvitaire is **depth-first search**
- But we need optimisations to avoid thrashing
  - Exploring endless possibilities that a bit more work can eliminate
- **Transposition Tables**
- **Exploiting Symmetry**
- **Streamliners**
- **Dominances**

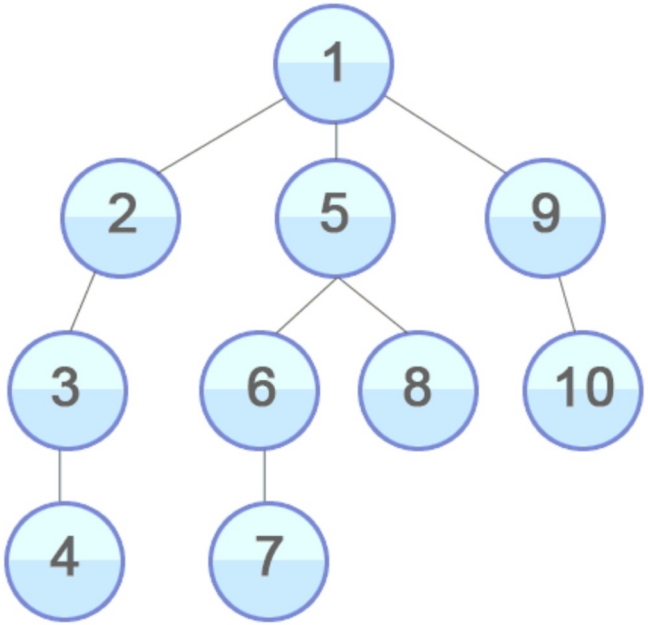
Depth First Search

- The core of Solvitaire is depth-first search
- We really just trust it to do total exploration
  - Only very mild heuristics
  - We don’t try to combine moves into “metamoves”



Down the rabbit hole

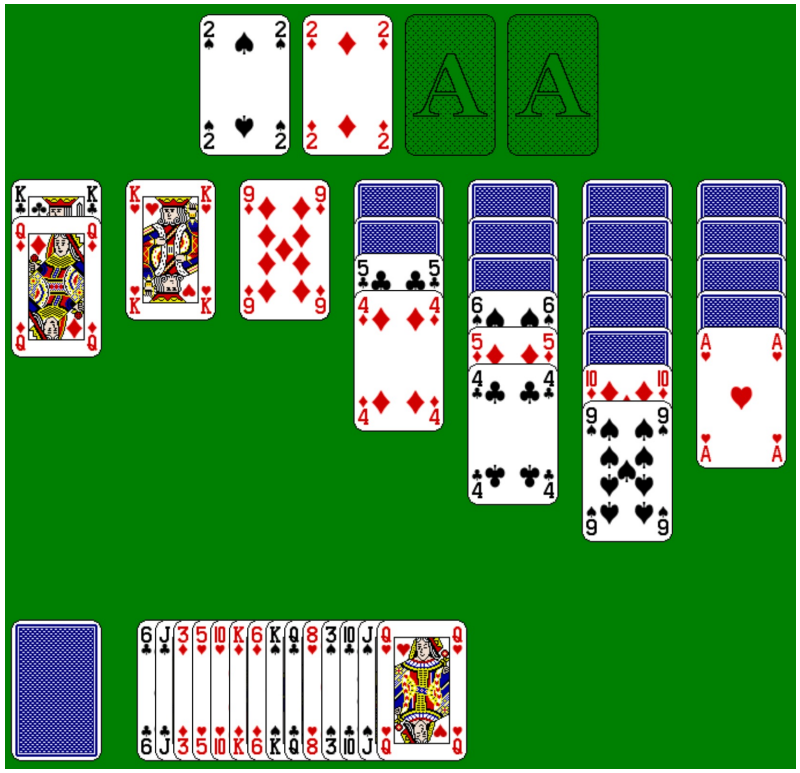
- Sometimes depths get ridiculous
  - One unwinnable instance was proved so at a depth of 27 million and searched about 1 billion nodes
  - Effective branching rate of about 1.0000008
- Certainly does go down a deep rabbit hole
- But it is able to explore the entire rabbit warren

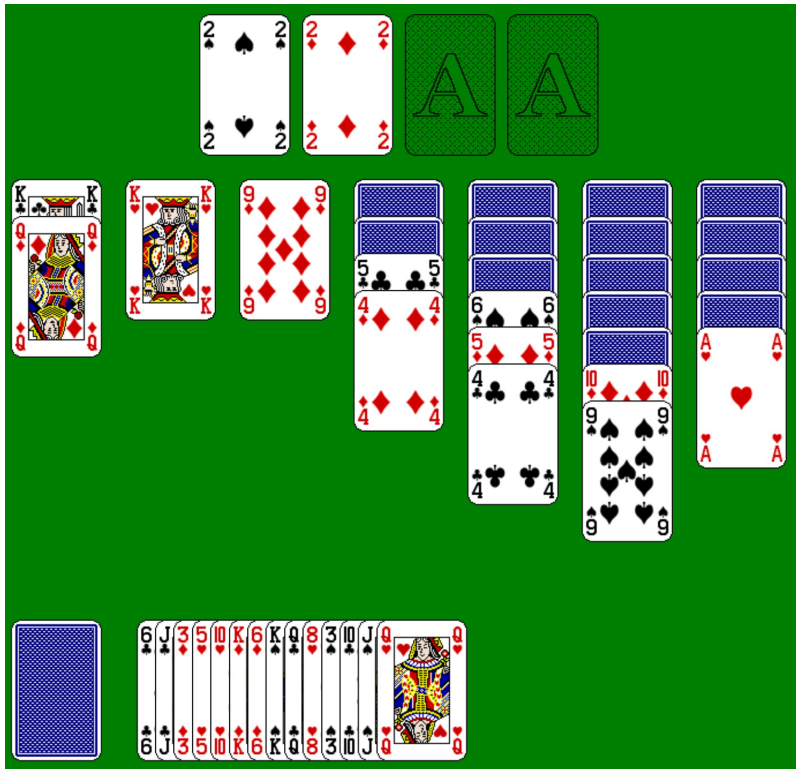


Implementation of Depth-First Search

- Solvitaire is written in C++
- Implementation of DFS is by “Trailing”
  - There is only one complete search state
- Every move destructively changes state
  - But we also put the move on a “Trail”
  - On backtracking pop moves from the trail
  - And each move can be reversed
- Every node in the tree checks the rules and current game state
  - This does mean pointless code is executed at every node
  - But doesn’t seem to have stopped our success

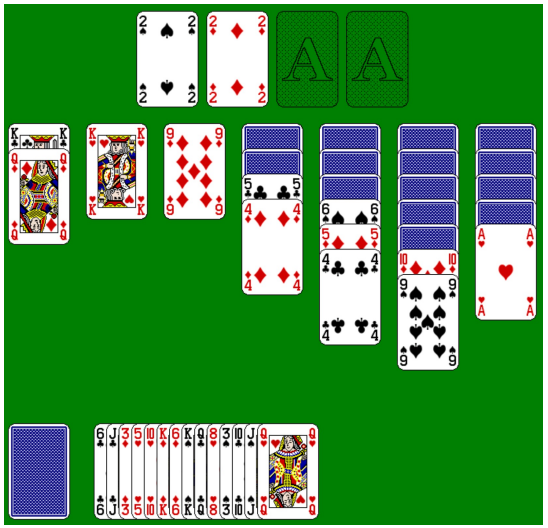
Transposition Tables:
Don’t try a position you’ve tried before!

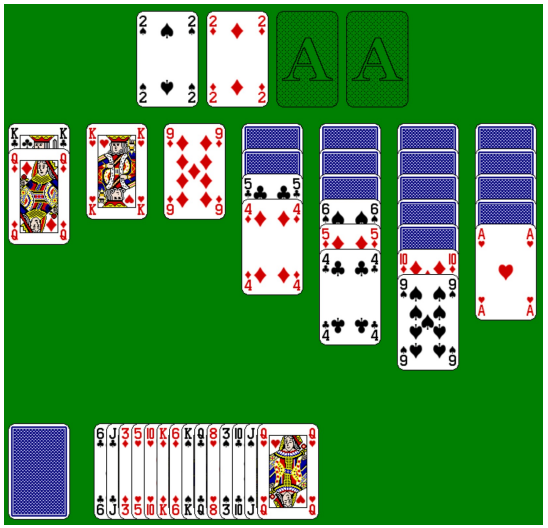




Transposition Tables

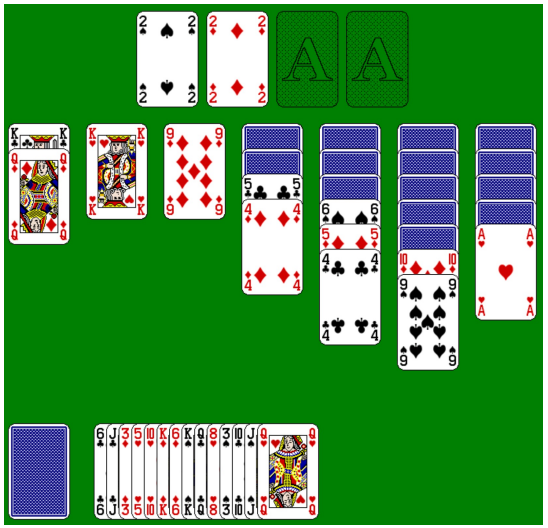
- If we have previously seen the top state in search, …
  … give up if we see the bottom state
- If we tried it before and backtracked
  - we must have failed
- If we tried it before without backtracking,
  - we have created an infinite loop, which is just as bad
- So use a cache of previously visited states
  - Standard in Games AI
- We do have to limit cache size for RAM reasons

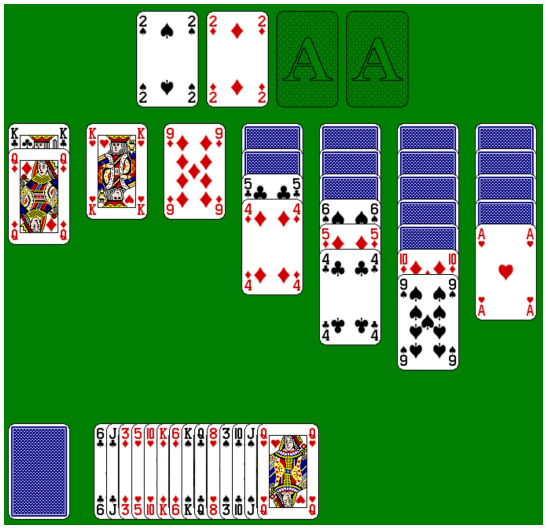




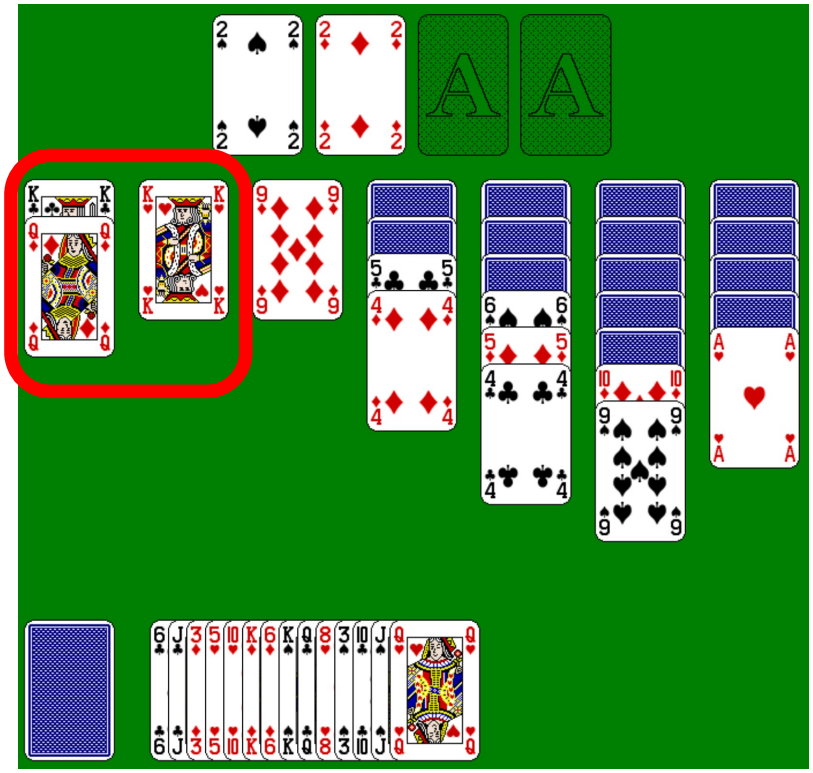
Transposition Tables

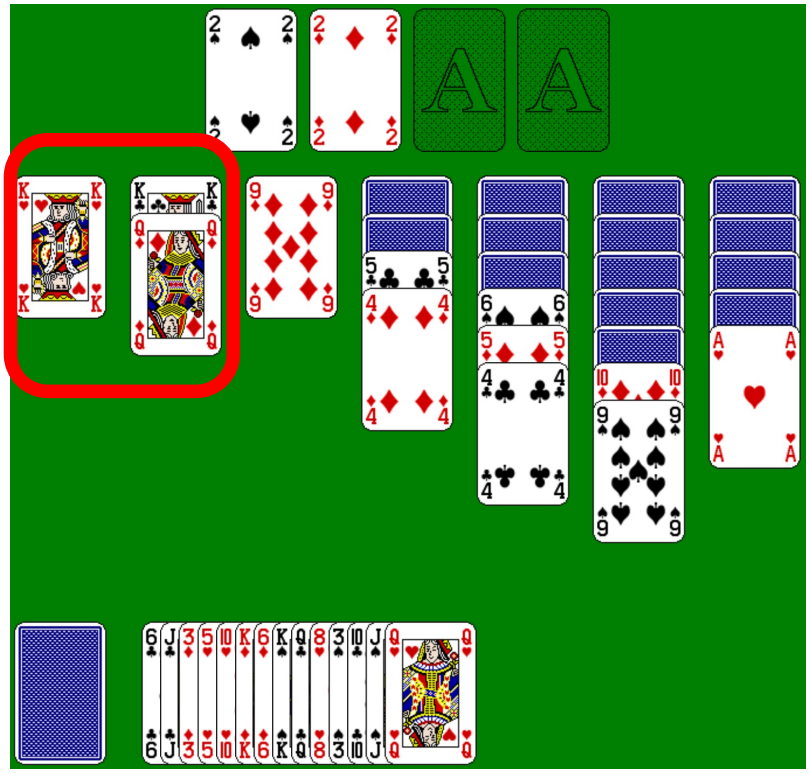
- Example in Klondike with 1 hour timeout
  - Cache size 100 million, 38 timeout of 10,000
  - Cache size 1 million, 492 timeout
- RAM is a problem though
  - E.g. with cache of 200 million, max RAM = 67 GB!





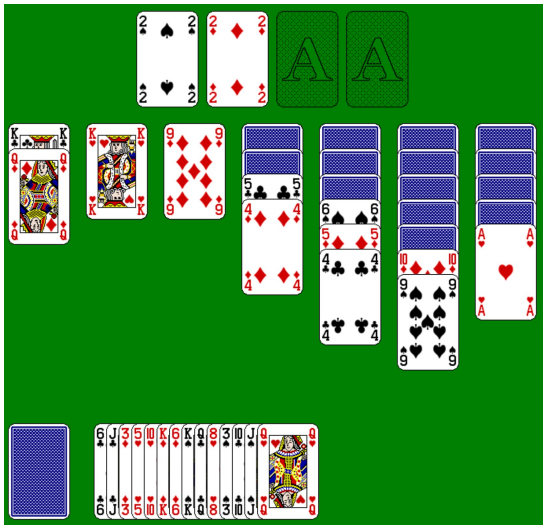
Symmetry Breaking:
Don’t try an equivalent position either!

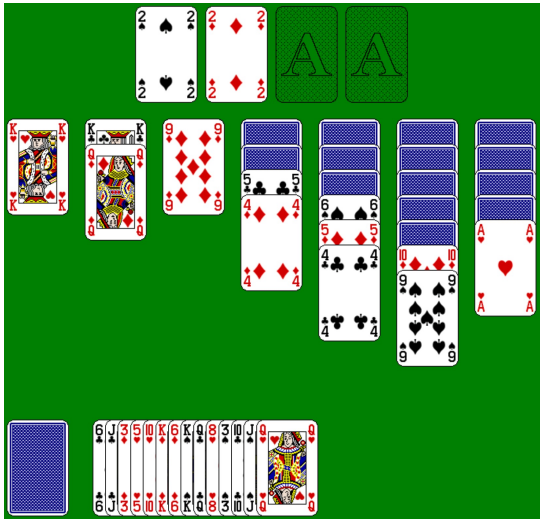




Symmetry Breaking

- As well as exact repetitions…
- Avoid a state symmetrically equivalent to a visited one
- E.g. the same piles in different columns
- We sort the columns in the tableau and that is what we store in the cache
  - i.e. we store a “canonical representative” of the state
- Example in Klondike with 1 hour timeout
  - Symmetry On, 38 timeout of 10,000
  - Symmetry Off, 195 timeout





Streamliners

- We try to “streamline” the search by hoping that a more restricted search might work
- The general idea is that when problems have solutions, they often have an unreasonably well structured solution
- In Patience, the idea is to make moves which are *usually beneficial*
  - Ignoring the fact that they *sometimes* force a loss
- If we win the game then we have won the game and all is golden
- If we don’t win, we don’t ***know*** that the game is unwinnable

Streamliners

- Most common example is:

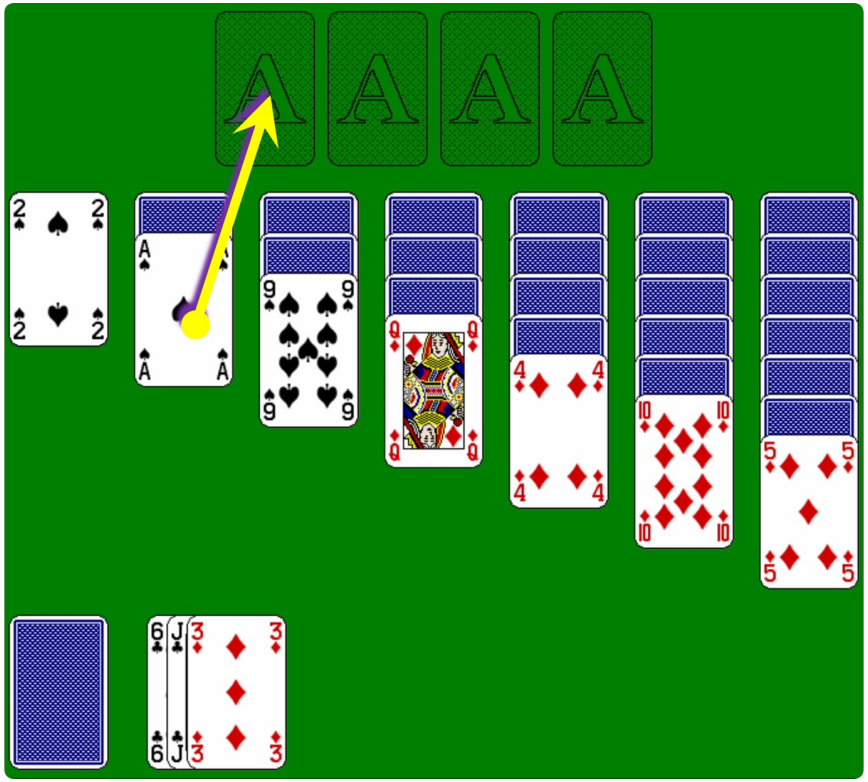
Always move a card to foundation if it is possible to do so

- Not making these moves will waste a lot of time
- But will sometimes make a winnable game unwinnable
- But most of the time it won’t
- And the reduced search space will find solutions MUCH quicker

Streamliners

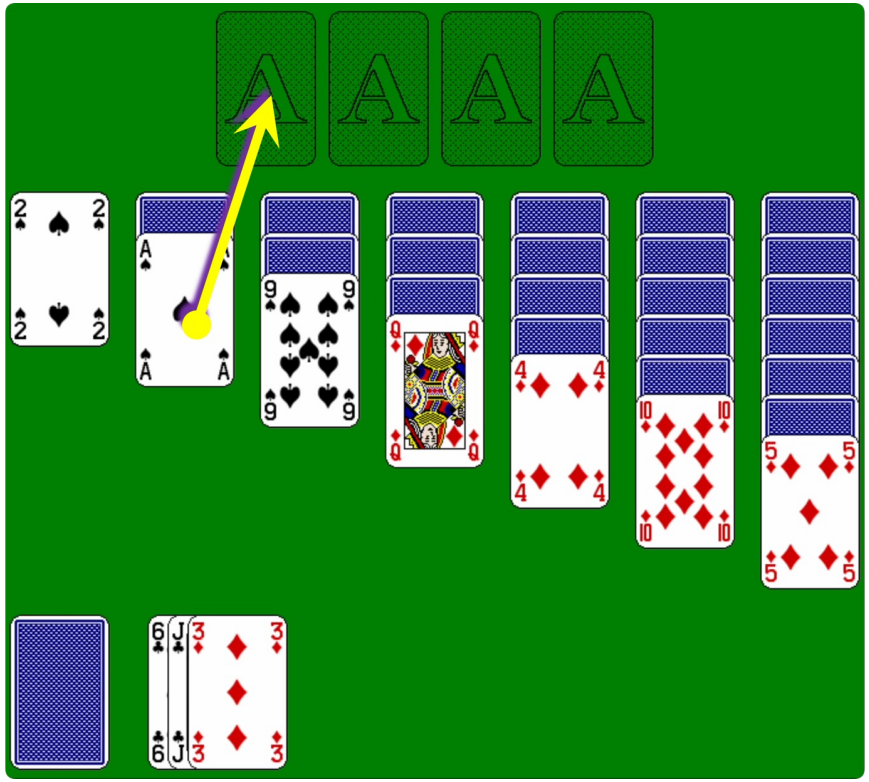
- What happens when a streamliner search says no?
  - We will have to rerun without the streamliners
  - And this will involve repeated search
- As a tradeoff we provide a “smart solvability” streamliner
  - run for 10% of allotted time with streamliners on
  - Then we start again 100% time if result indeterminate
- Example in FreeCell, 5 minute timeout
  - Smart Streamliner On, 0 timeout of 10,000
  - Smart Streamliner Off, 254 timeout

Dominances:
If you might as well make a move, make it!



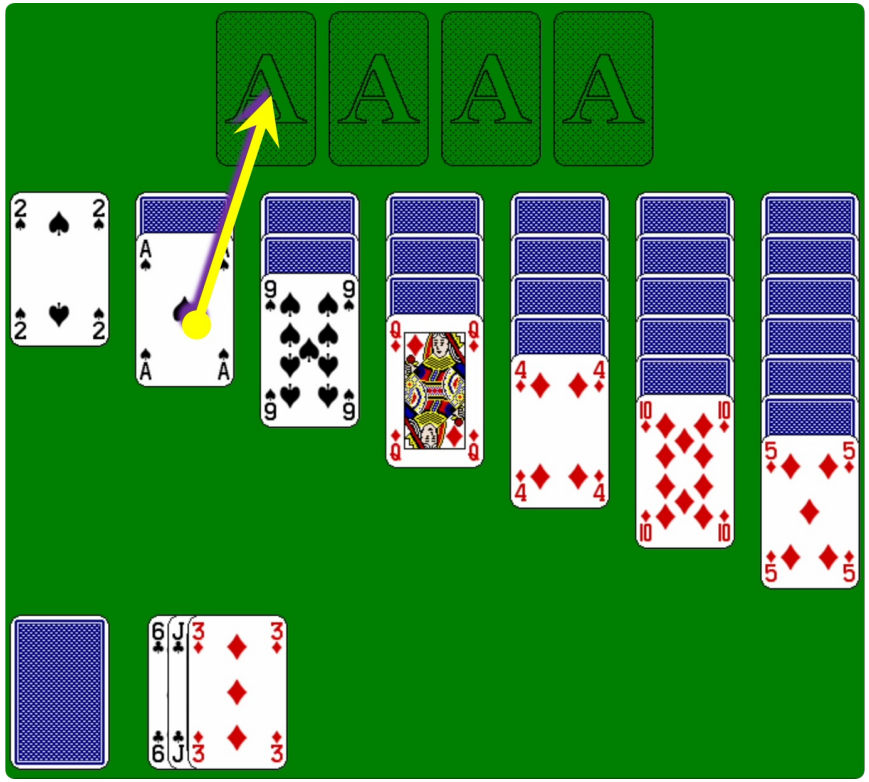
Dominances

- We might as well move the Ace to foundations
- We *could* wait to do it
- But there would have to be a reason
  - E.g. have to use the Ace
  - But there is nothing to use the Ace for
- So we don't *have* to do it
- So we might as well
- It's **safe** to make this move



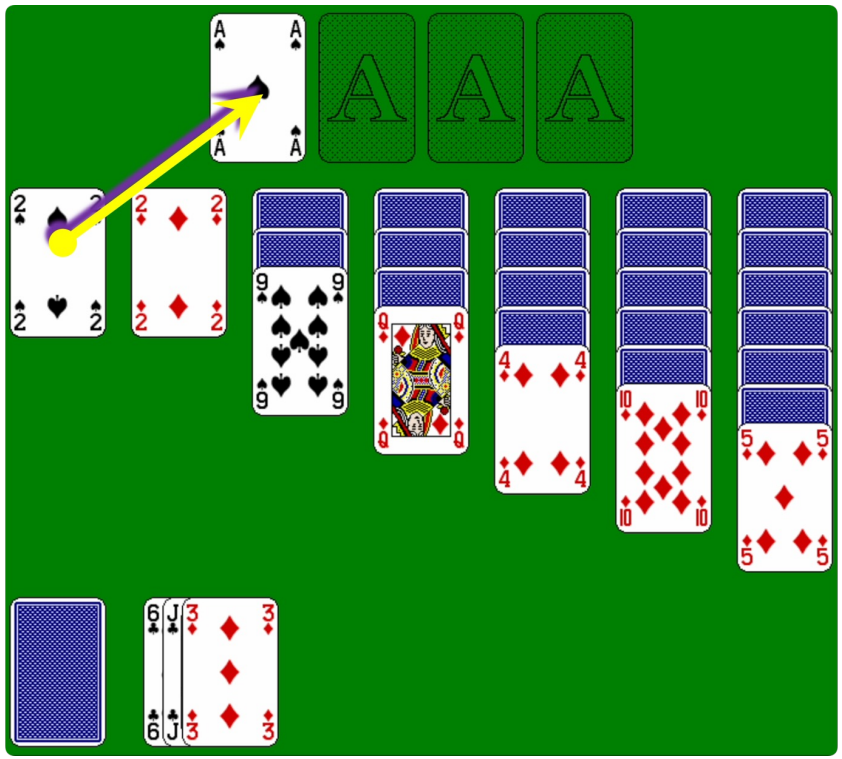
Dominances

- Sometimes called “Safe moves”
- A Dominant move is one that we can play safely knowing that
  - If there is ANY solution
  - there is one where this move is made now
- Dominances have been very widely used in patience solving
  - And are incredibly important (see later)
  - But an incredibly prolific source of bugs (also see later)



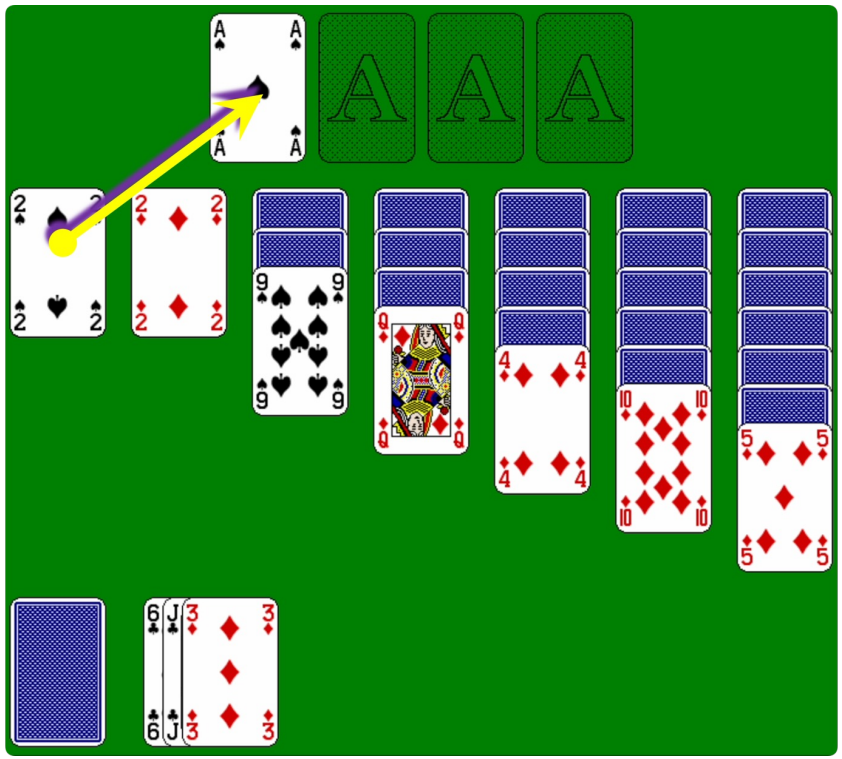
Dominances

- We might as well move the 2 too
- Here the 2 might be useful
  - You could build a card on it
  - Specifically a Red Ace
  - But …
    - There would be no need to
    - Because we could move the Ace to foundations instead
- This idea extends a bit further



Dominances

- The general rule in games like this is
- A card is safe to move to foundation if
  - the foundations of the opposite colour are both at most two below this card
  - **AND** the other foundation of the same colour is at most three below this card
- Called “clear and obvious rule” by Michael Keller
- Used in many solvers
- But nobody ever proved it’s sound
- So we did

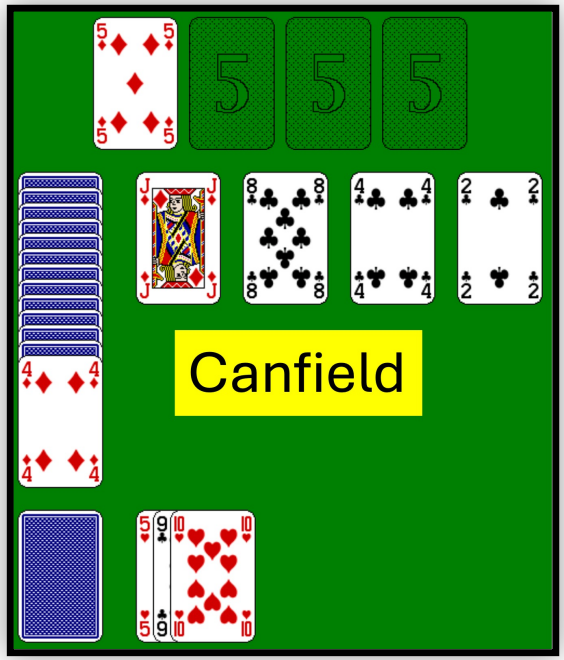


Fun Fact

- We didn’t prove this in original version of paper
- Reviewers said we should, and of course they were right
- Fun fact: Reviewers can be right and help make your paper better!

Testing and Finding Bugs

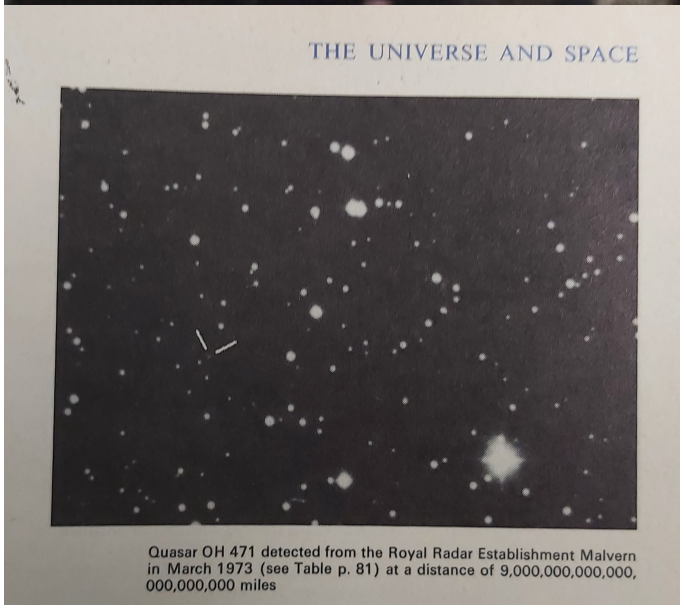
- We were able to find bugs in Solvitaire and in Wolter's code
- Using comparisons at many levels
- **Macroscopic:**
  - we got inconsistent estimates
- **Microscopic Level**:
  - different results for same instance
  - one solver would make an illegal move
  - or refuse to make a legal one
- Required "Punctilious Tenacious Precision"



Fun Fact

- My father wore a monocle …
- … but saw further than anybody else
- Finding the most distant object in the universe
  - As of 1973 anyway
- His group's attitude in Radio Astronomy was

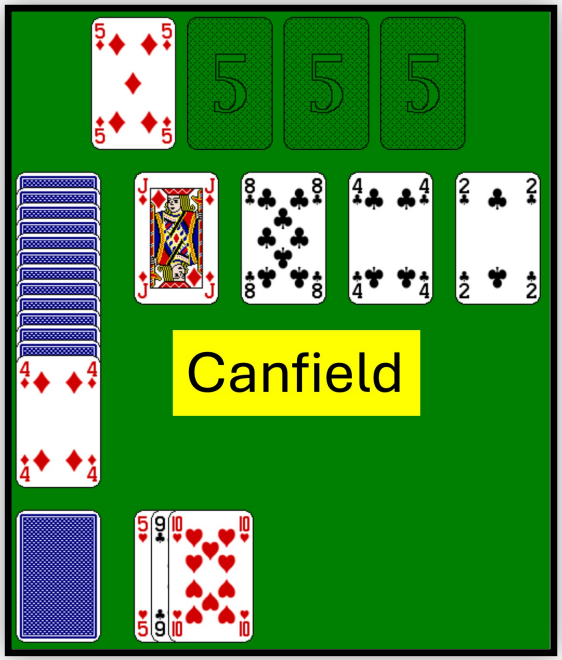
"Punctilious Tenacious Precision"



Quasar OH 471 detected from the Royal Radar Establishment Malvern in March 1973 (see Table p. 81) at a distance of 9,000,000,000,000,000,000,000 miles

Punctilious Tenacious Prevision

- Many iterations of finding bugs in both Solvitaire and Jan Wolter's solver
  - For Canfield but would be the same in Klondike
- Here is the most ridiculously complicated one
- Wolter had a dominance that turned out to be wrong
- Move card to foundation when the **penultimate card** in the stock satisfied the conditions
- This is incredibly close to correct but not quite there



Punctilious Tenacious Precision

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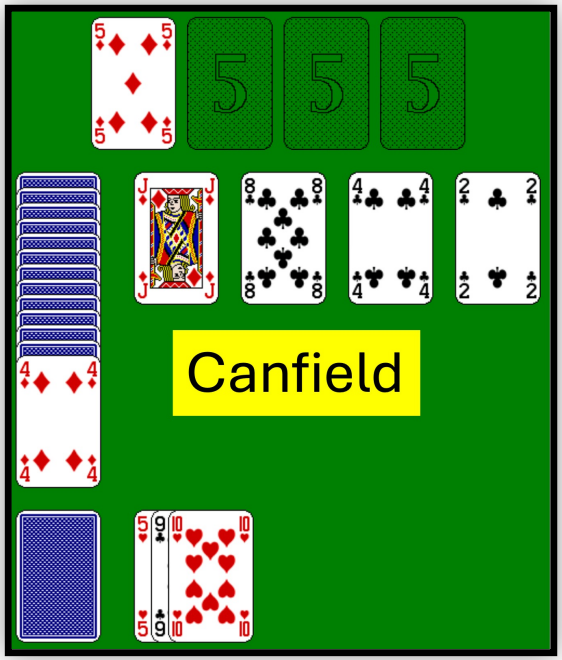
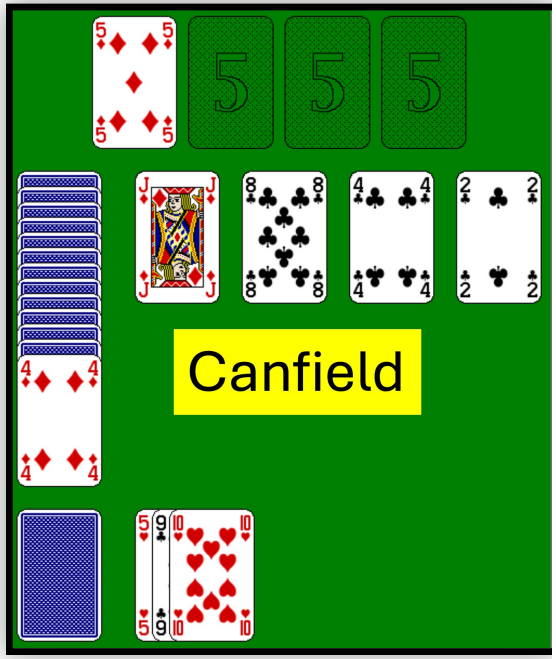


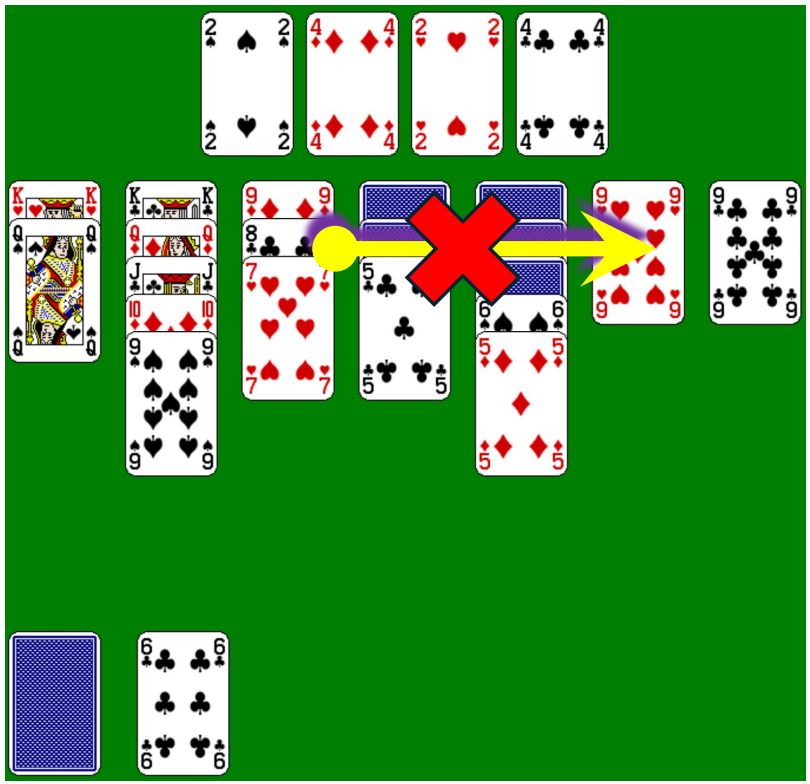
Punctilious Tenacious Precision

- If you play the penultimate card in the stock ....
  - Then much later on
  - You play the third card in the stock
  - Revealing the second card in the stock
  - Which you must not play immediately
  - But if you don't play immediately you can't ever play it
- If you had *not played* the penultimate card in the stock
  - And now you can leave it as a placeholder
  - And when you play the third card in the stock ...
  - .... it's eventually replaced by the second last
  - And the two conditions are now simultaneously achievable
- And we DO see random deals where this makes the difference between winnability and unwinnability



NOT finding bugs in Canfield

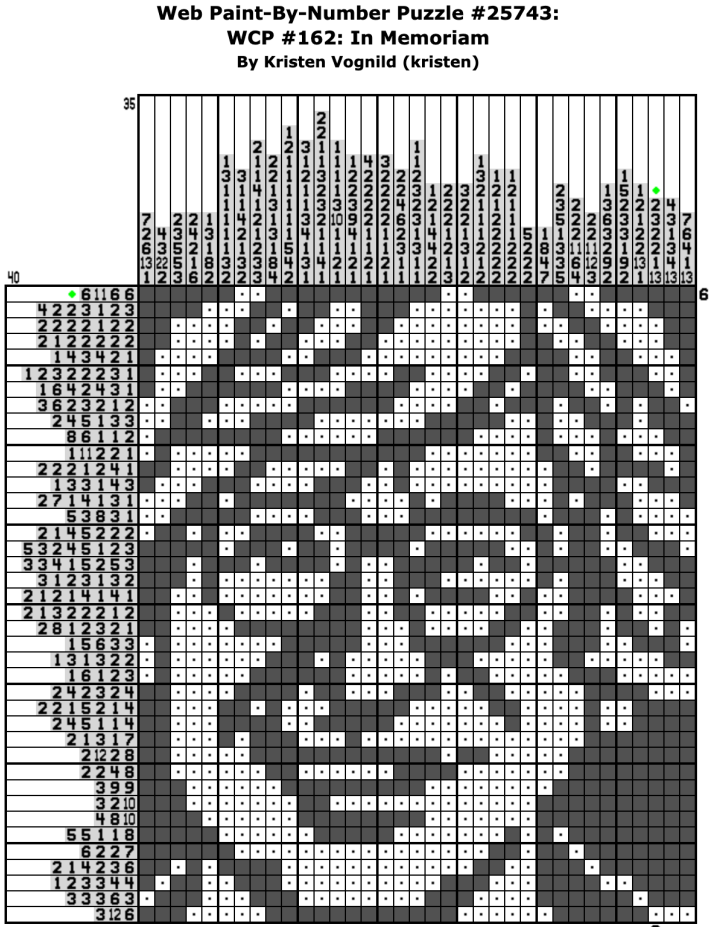
- We thought we had found a similar bug
- But never seemed to change results
- **ONLY ALLOW** a partial pile move
  - **IF** the card currently covering the top card being moved
  - ***CAN BE PLAYED TO FOUNDATION***
- But it is in fact correct
  - So we **generalized it** and we **proved** it
- Example in Klondike with 1 hour timeout
  - Dominances On, 38 timeout of 10,000
  - Dominances Off, 481 timeout



Not so Fun Fact

- Jan Wolter died 1/1/2015
- But before he did …
- … he put all his solvers' code online
- And it's open source and still there
- And its availability helped us …
  - Reproduce his results
  - Debug Solvitaire
  - Use a dominance we found in his code

Open Source: Let Your Code Outlive You!



What if we change the rules?

- Not being hardwired, we can easily change the rules
- What if we try Klondike with a different set of rules?
- Solvitaire can do this …. Usually

Spaces Policy \ Build Policy	Any Suit	Red-Black	Same Suit
Any	$99.923 \pm 0.006\%$	$94.959 \pm 0.045\%$	$40.762 \pm 0.097\%$
King Only	$99.855 \pm 0.049\%$	$81.945 \pm 0.084\%$	$6.895 \pm 0.050\%$
Not allowed	$\mathit{51.135 \pm 48.759\%}$	$2.168 \pm 0.121\%$	$0.178 \pm 0.009\%$

What if we change the rules?

- Not being hardwired, we can easily change the rules
- What if we try Klondike with a different set of rules?]
- Solvitaire can do this …. Usually **but not always**

Spaces Policy \ Build Policy	Any Suit	Red-Black	Same Suit
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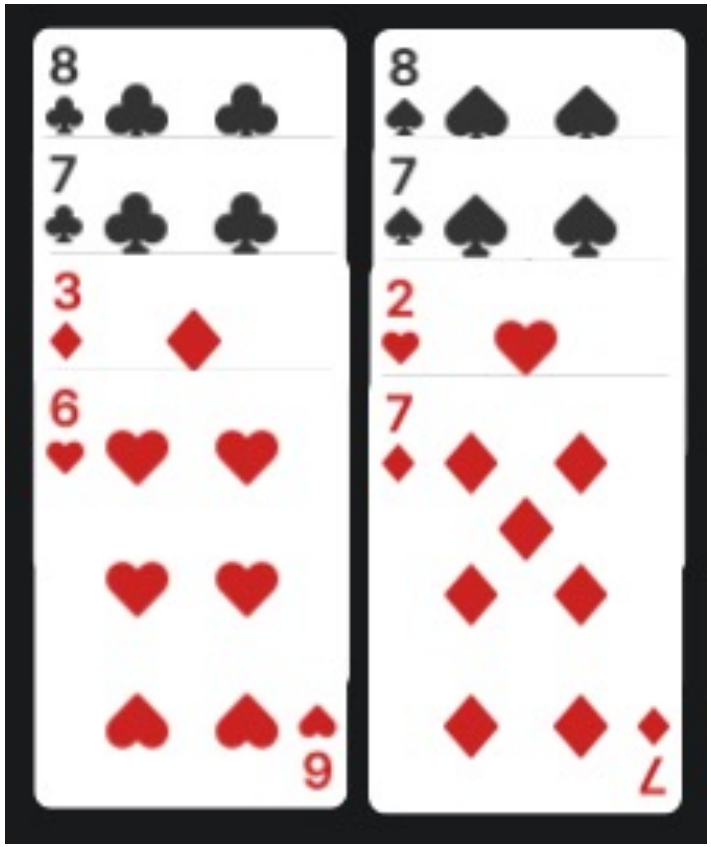
Conclusions

- The most famous single player card game is called “Klondike”
- Single player card games are called “Solitaire” or “Patience”
- There are hundreds of other Patiences
- We built a solver “Solvitaire” which can solve many many of these
- Solvitaire got world’s best results on dozens of Patience games
- Uses just a basic JSON description of rules
- It is a classic example of “Good Old Fashioned AI”
  - Depth-First Search with some GOFAI add-ons
  - Transposition Tables, Symmetries, Dominances, Streamliners
- And Solvitaire isn’t perfect so we can improve results with Constraints

Blocked Positions

- Solvitaire doesn't use constraints in any way
- And examples like this cause problems
- *Imagine all but the bottom cards are hidden*



- We can't **ever move** the bottom two cards
- They are **blocked**

- But Solvitaire can thrash failing to solve positions like this

Constraints in Patience Games

- Constraints have been used for some patiences
- But typically ones with a definite move count
  - E.g. each card moves exactly once
- It’s hard to come up with good models for many other games
  - They often have complex rule sets
  - We don’t know how many moves will be needed
- For Klondike we have ***not*** got a complete model

Constraints in Klondike

- For full details see Jack Waller's wonderful talk .... Yesterday
- We solve a *relaxed* version of Klondike
- But unwinnable layouts in the relaxed version are unwinnable in the full game
- And we are able to improve our estimate of winnability
  - By proving some layouts unwinnable
- **Solvitaire gave:** 81.945 ± 0.084 %
- **DRUM ROLL....**

Constraints in Klondike

- For full details see Jack Waller's wonderful talk .... Yesterday
- We solve a *relaxed* version of Klondike
- But unwinnable layouts in the relaxed version are unwinnable in the full game
- And we are able to improve our estimate of winnability
  - By proving some additional layouts unwinnable
- **Solvitaire gave:** 81.945 ± 0.084 %
- **DRUM ROLL....: 81.942 ± 0.081 %**

Breaking News

- Not in our ModRef paper…
- Constraints prove **72%** Any Suit/Not Allowed games unwinnable
- So they do show a lot of promise

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- One huge open question remains

The Big Open Question

- With best possible play…
- How winnable is Klondike when you don’t know where the cards are?

- Incredible range of uncertainty
  - It is *definitely* less than about 82%
  - It *might* be at least 43%

- We might never know this as we might not be able to *prove* that a method of play is best possible

And finally…

The Most Amazing Fun Fact

- We use Monte Carlo methods to estimate winnability
- Monte Carlo Methods were invented by Stanislaw Ulam
- ….

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The Most Amazing Fun Fact

- We use Monte Carlo methods to estimate winnability
- Monte Carlo Methods were invented by Stanislaw Ulam
- ….
- When he lay in bed convalescing and playing “Canfield”
- He wanted to know how winnable it was
- Analysis seemed too hard
- So thought of randomly laying out deals and computing how many came out
- **We have now used Monte Carlo methods to do exactly what their inventor conceived of them doing**

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Acknowledgements

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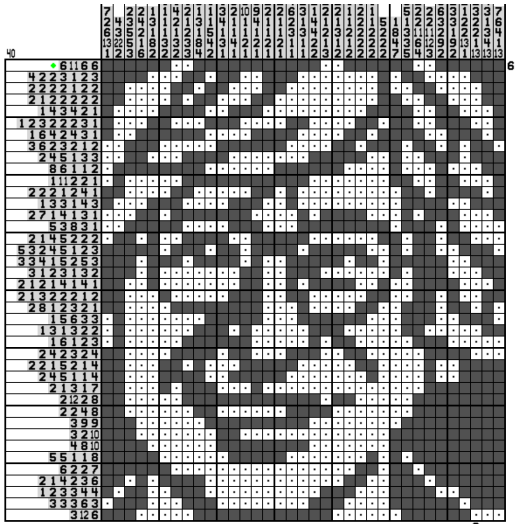
And Posthumously...

Jan Wolter, Hubert Gent, Margaret Gent



Any Questions?