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The Constrainedness of Arc Consistency

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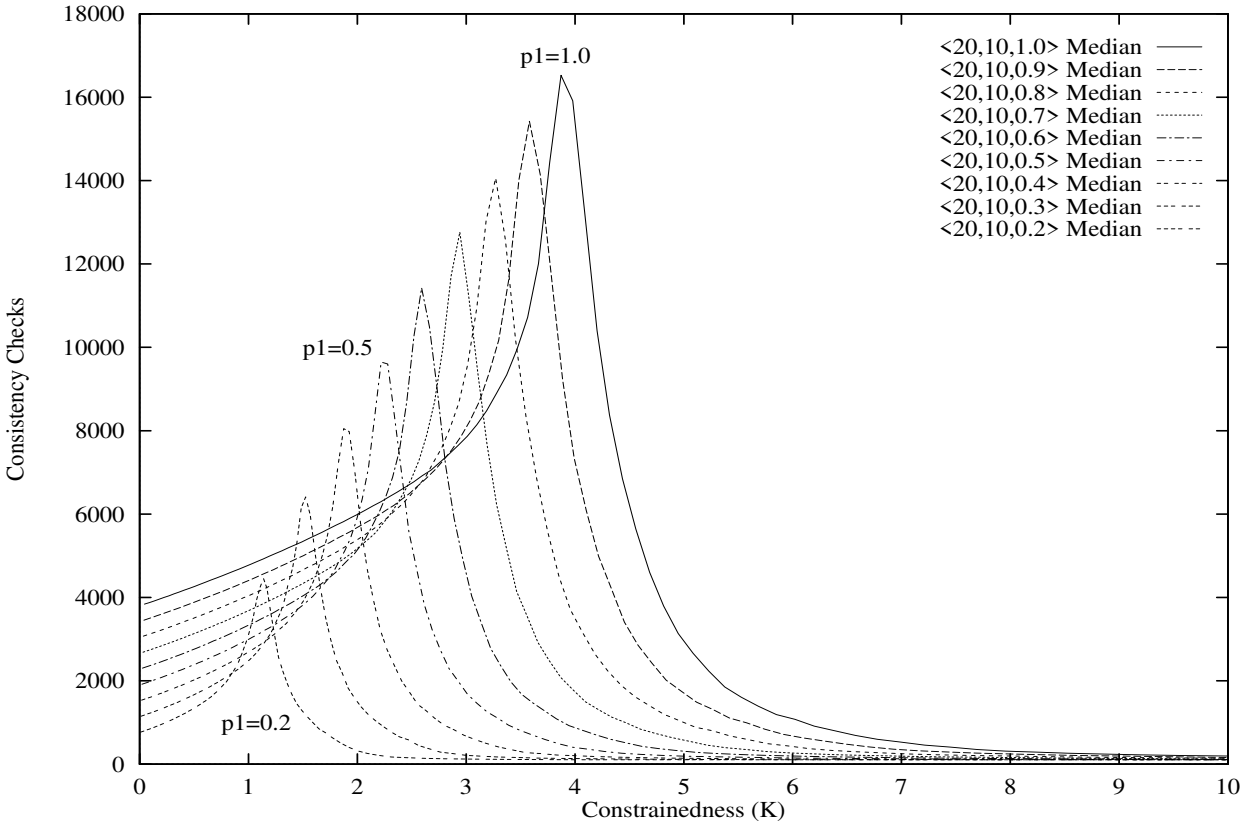
Started here!

CP96 Best Poster Award



Contours march along the x-axis

Can we fix them?

Why bother?

- What is the AC decision problem?
  - for each variable, is there a (non-empty) subset of its domain such that each value is supported?
- The obvious way
  - apply AC3/4/5/6/7, if DWO then NO else YES
  - problem is in P (cubic worst case)
- The really daft way
  - search: assign each variable a subset of its domain and ask "*Are all values supported*?"

$$\kappa = 1 - \frac{\log_2(< Sol >)}{N}$$

<Sol> is expected number of solutions

N is log_2 of the size of the state space

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k = 0, all states are solutions, easy, underconstrained

k = ∞ , <Sol> is zero, easy, overconstrained

k = 1, critically constrained, 50% solubility, hard

Applied to: CSP, TSP, 3-SAT, 3-COL, Partition

- Search for a state with required property
- What is a search state?
  - each variable assigned a subset of its domain
- How many search states (I.e. what's *N*?)
- What is probability state has required property (I.e. what's <*Sol*>?)

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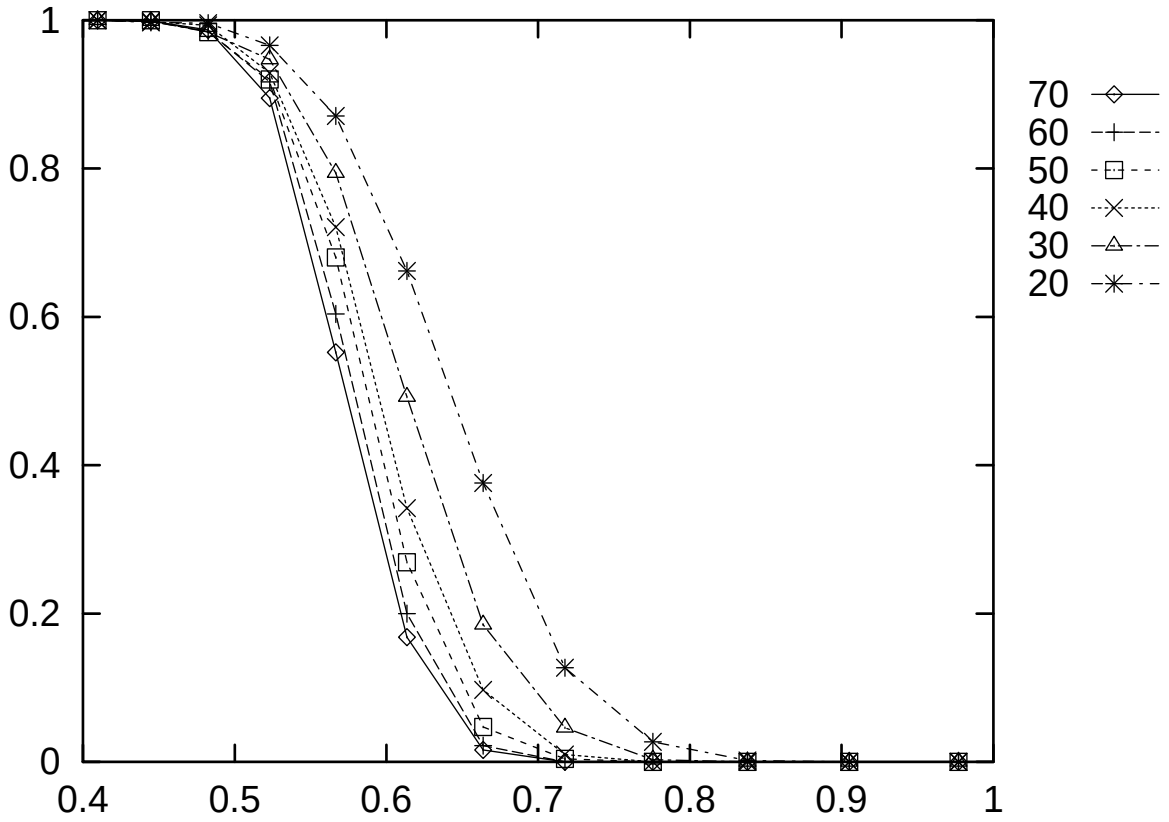
Answer all of these questions and we get

$$\kappa_{ac} = -\frac{1}{2} p_1 (n-1) \log_2 (1 - p_2^{m/2})$$

for uniform problems, where <n,m,p1,p2> is ...

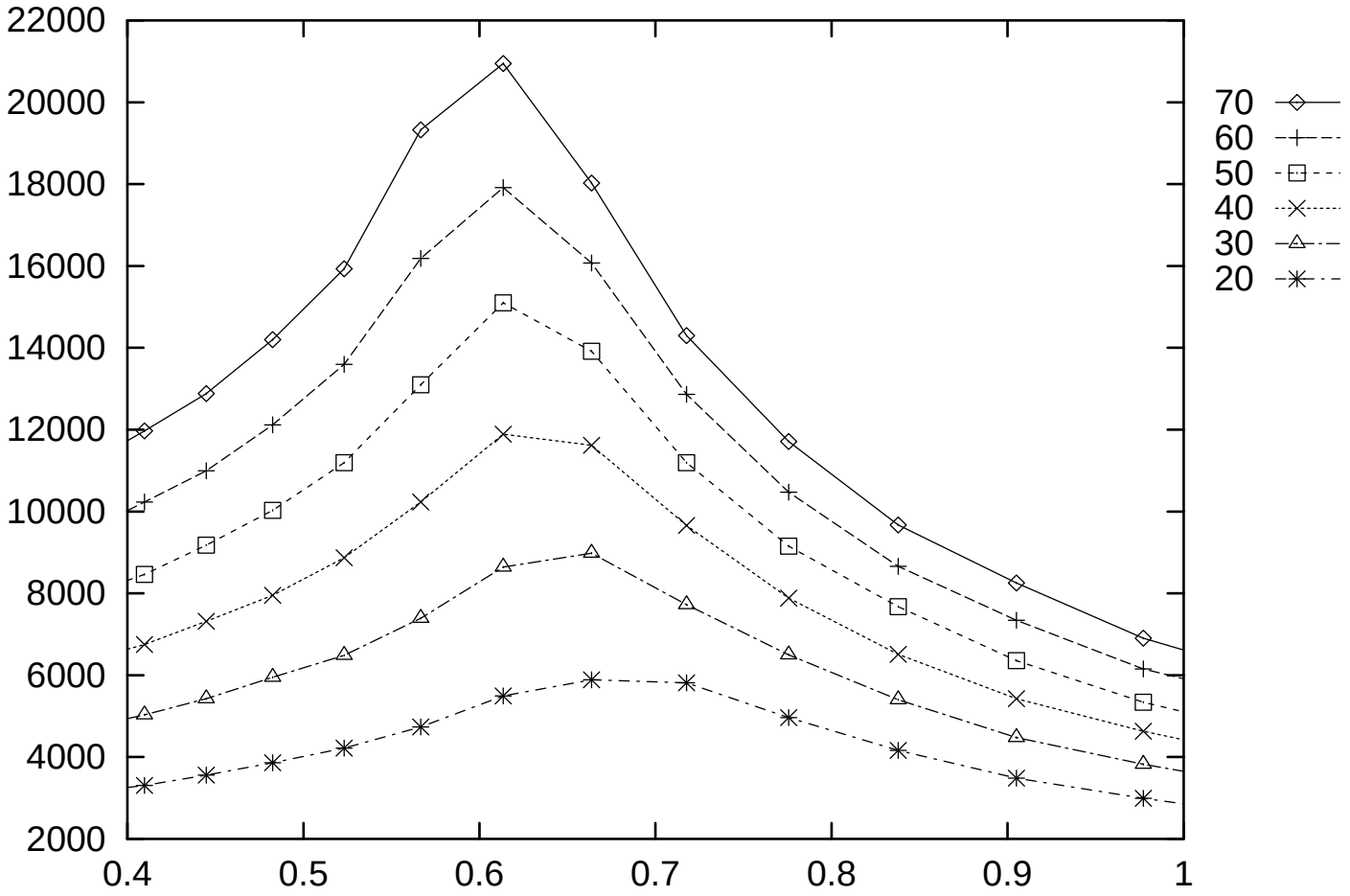
[For non-uniform see the paper]

Solubility phase transition in AC



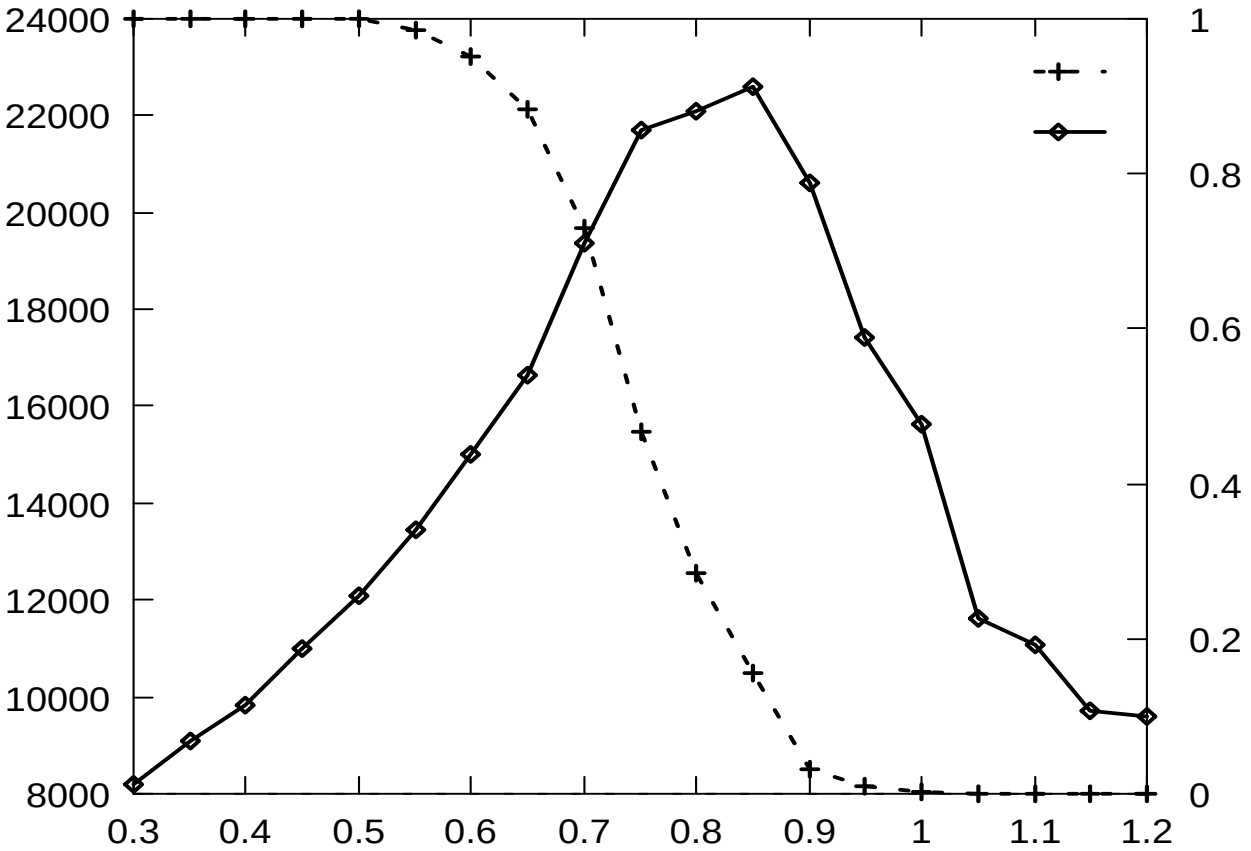
Probability of establishing AC (x) against k_ac
(number of variables 20, 30, … 70)

The complexity peak



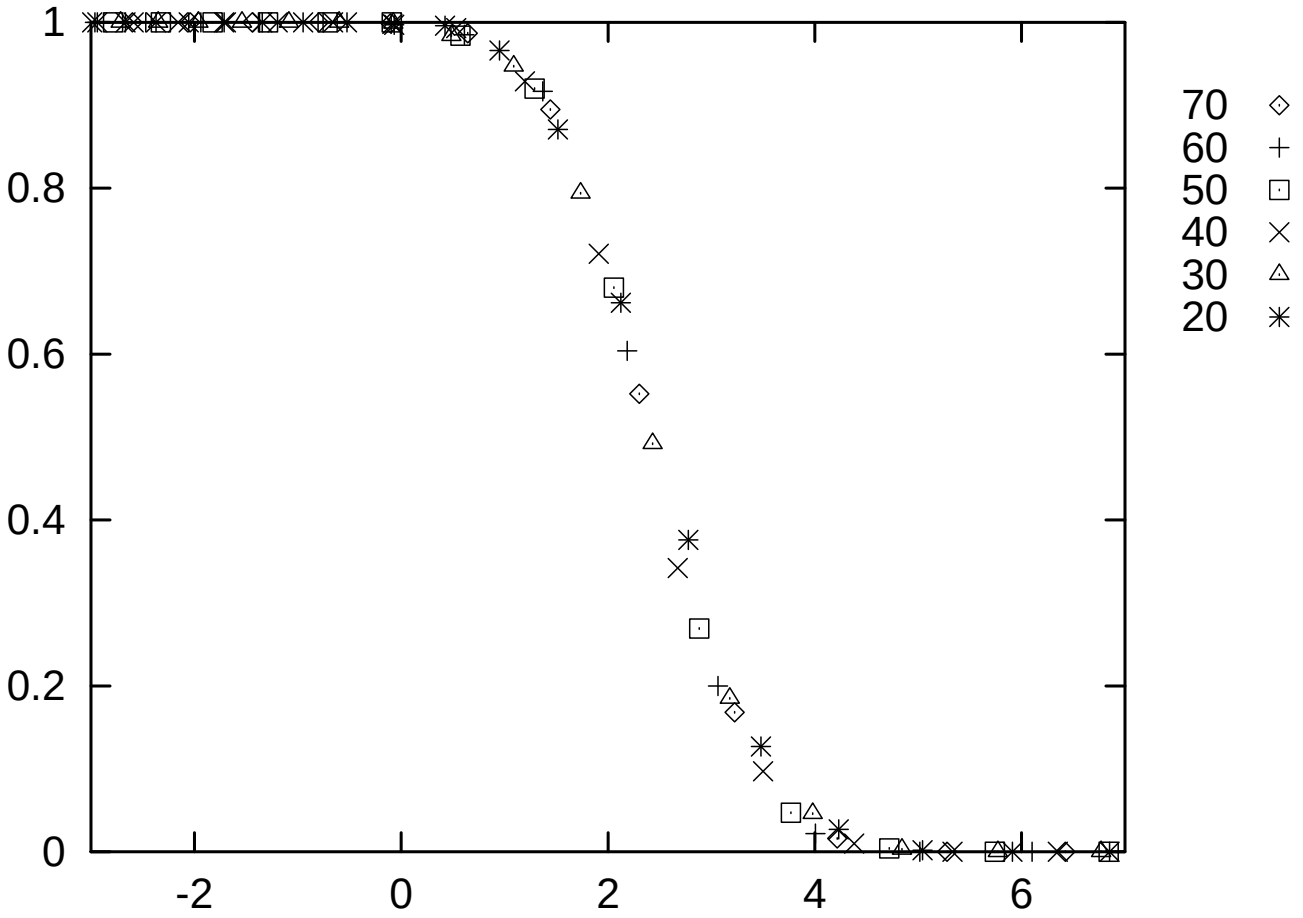
Median consistency checks for AC3 against k_ac (again, 20 to 70 variables)

non-uniform problems (m = 10 or 20)



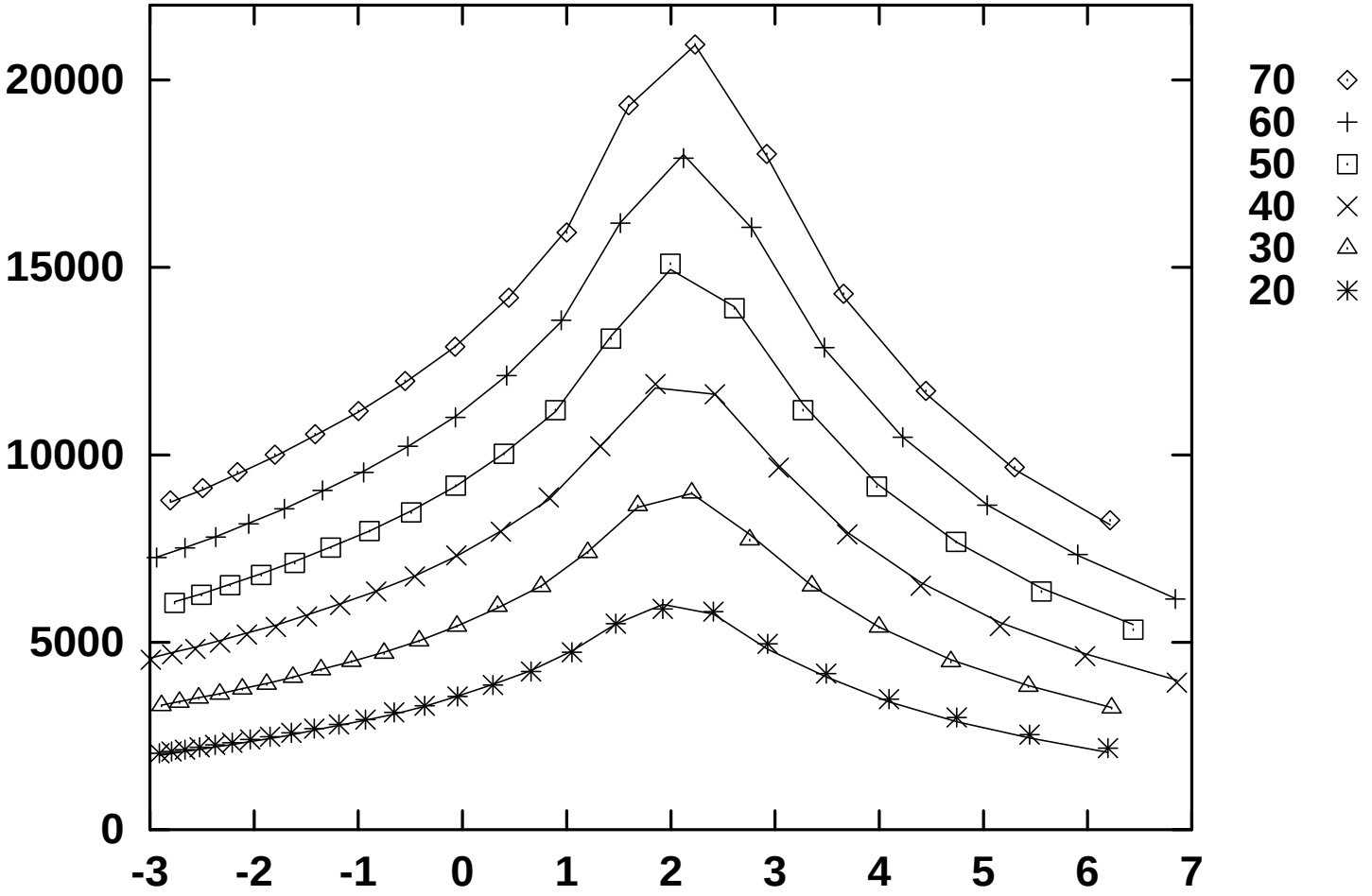
Schiex says " There is no clear wipe-out threshold as in the usual fixed domain size model"

Finite size scaling



Rescaled solubility phase transition (gamma on x-axis)
(20 to 70 variables)

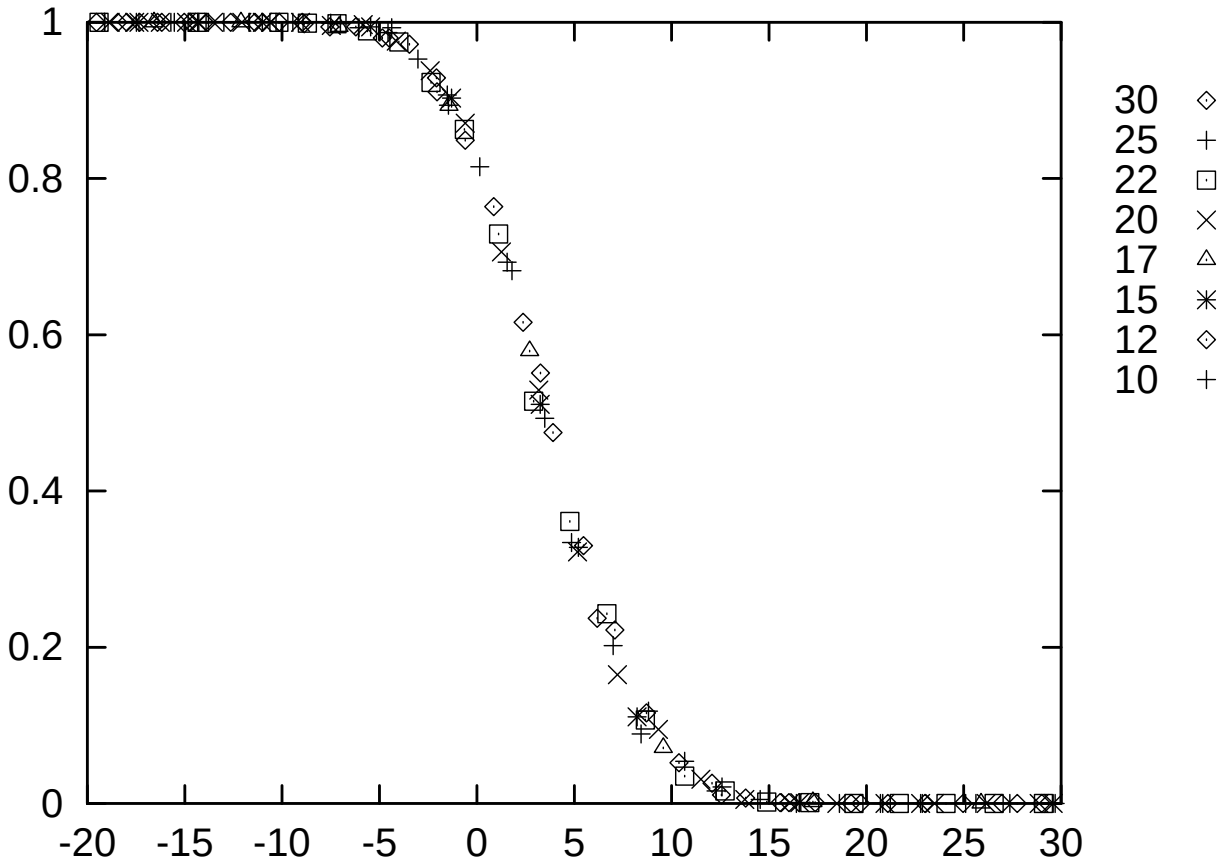
Finite size scaling



Median AC3 checks, gamma on x-axis

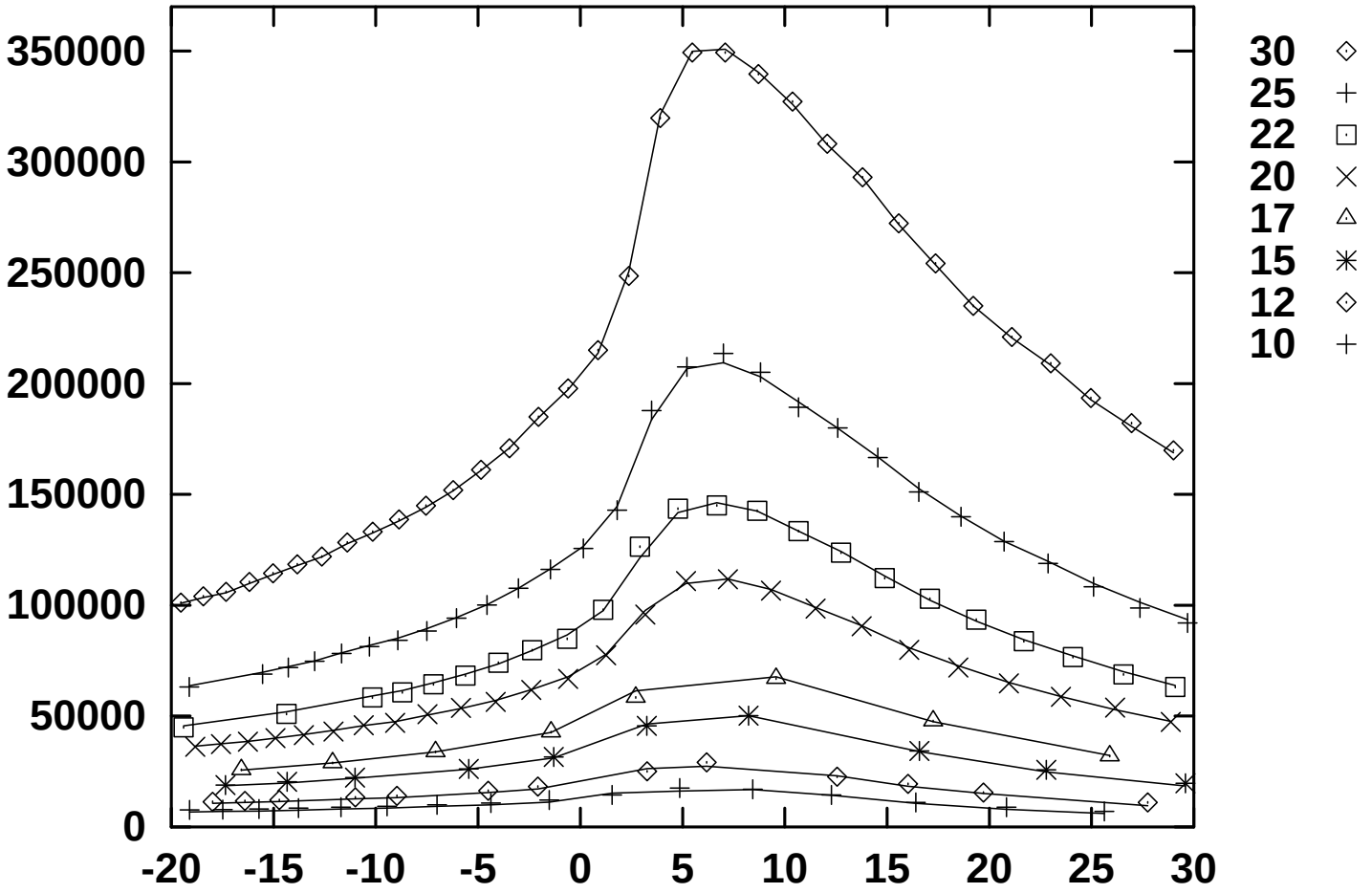
Points observed, lines predicted

Finite size scaling



Rescaled solubility phase transition, varying domain size (domain size 10, 12, 15, .., 30)

Finite size scaling

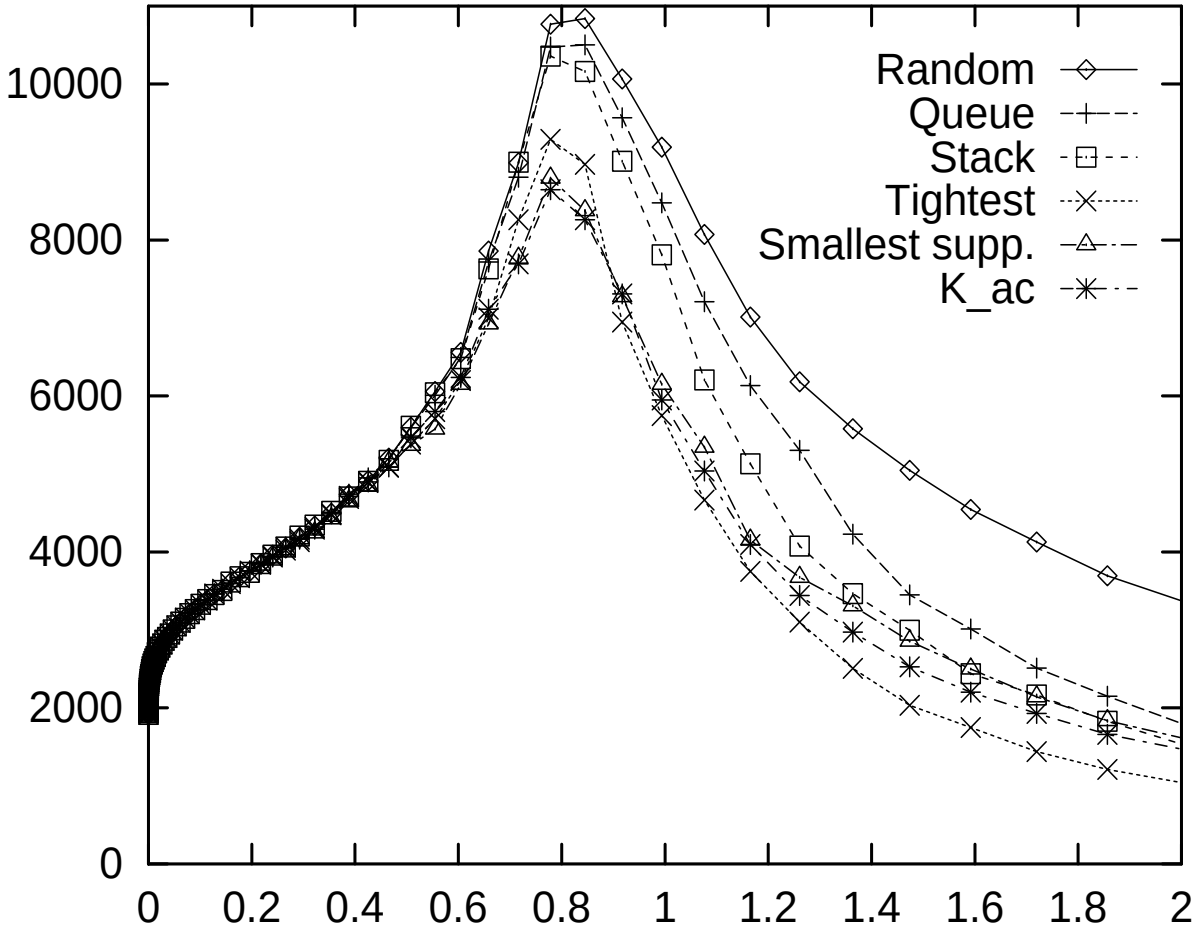


Median AC3 checks, varing domain size

Points observed, lines predicted

- Minimise constrainedness
  - when *making a choice*, leave the *remaining subproblem* as unconstrained as possible
- Applied to AC
  - select a constraint to revise
  - remaining subproblem is defined by the set of constraints remaining to be revised

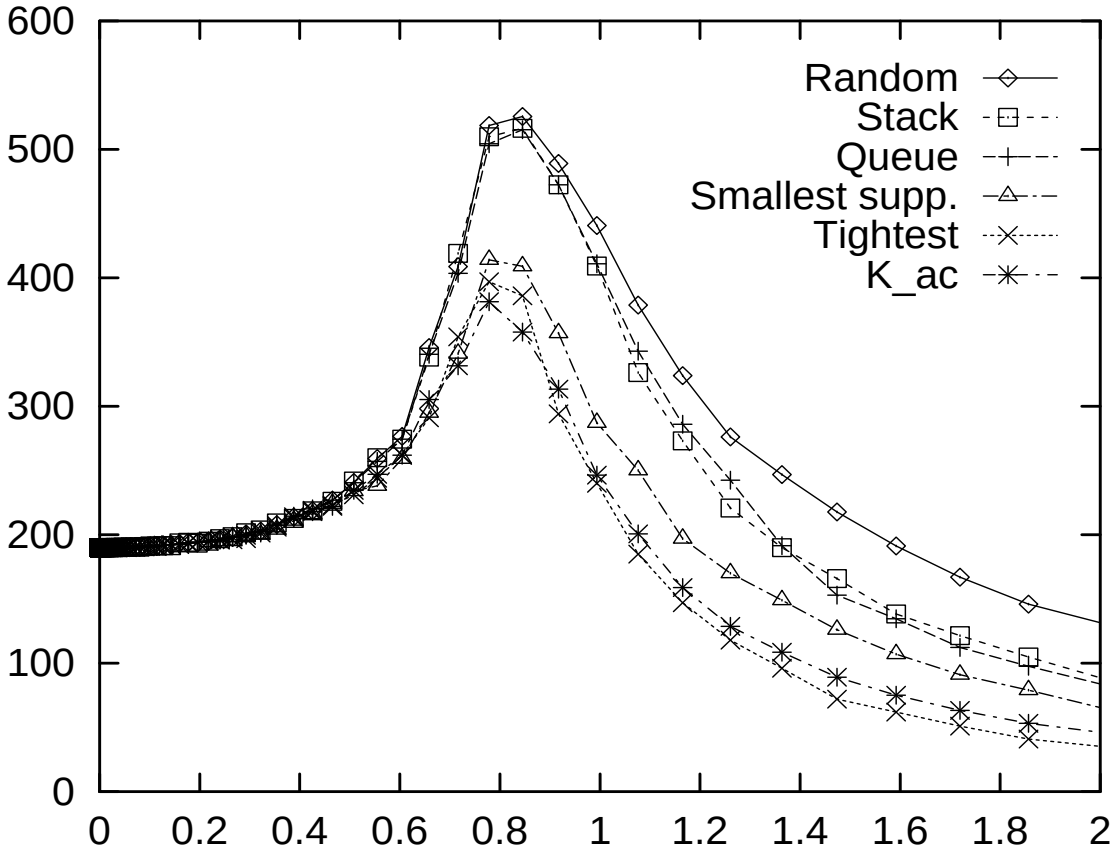
As a heuristic



Median AC3 checks, various constraint ordering heuristics

… including minimise constrainedness

As a heuristic



Median AC3 revisions using various constraint ordering heuristics

…. including minimise constrainedness

Conclusion

- We've carried constrainedness into P
  - locate phase transition
  - finite size scaling, predict costs
  - heuristics, constraint ordering (use in MAC?)
- AC3 in 1977, complexity in 1985
  - what if it were 1984?
  - Is there a 1990's equivalent?
- Have a good look at CP97 posters!

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Odd one out?