

This talk is based on joint work with Bogdan Alecu, Robert Ferguson, Mamadou Moustapha Kanté, Vadim Lozin, Sean Mandrick, and Viktor Zamaraev

Geometric grid classes of permutations [1], are defined using matrices whose entries indicate monotone segments in specified grid cells. Given a matrix, its geometric grid class includes all permutations whose plots can be “drawn on” the corresponding grid figure.

Lettericity, introduced by Petkovšek [2], is a graph parameter defined via representations known as letter graphs. A letter graph is constructed from a word over a finite alphabet together with a decoder—a specified set of letter pairs determining adjacency. Vertices correspond to letters of the word, and edges between vertices are dictated by the decoder. The lettericity of a graph is the smallest alphabet size needed for such a representation.

Though initially appearing unrelated, geometric griddability and lettericity share deep structural similarities. This talk presents results from Alecu, Ferguson, Kanté, Lozin, Vatter, and Zamaraev [3] demonstrating that a permutation class is geometrically griddable if and only if the associated graph class has bounded lettericity. The discussion will also include results by Mandrick and Vatter [4] on the typical lettericity of graphs: almost all graphs on n vertices have lettericity between $n - \frac{1}{2} \log_2 n$ and $n - (2 \log_2 n + 2 \log_2 \log_2 n)$. Finally, the talk will conclude by contrasting this result with the analogous problem for permutations.

REFERENCES

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- [3] Alecu, B., Ferguson, R., Kanté, M., Lozin, V. V., Vatter, V. R., and Zamaraev, V. (2022) Letter graphs and geometric grid classes of permutations. *SIAM J. Discrete Math.*, **36**(4), 2774–2797.
- [4] Mandrick, S. and Vatter, V. (2024) Bounds on the lettericity of graphs. *Electron. J. Combin.*, **31**(4), Paper No. 4.53, 8 pp.