CHARACTERIZATION OF AVOIDANCE OF ONE-SIDED 2- AND 3-SEGMENT PATTERNS IN RECTANGULATIONS BY MESH PATTERNS

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joint work with Andrei Asinowski,[2] Namrata[3], and Torsten Mütze[4]

Permutation Patterns 2025
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July 9, 2025

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[2] Alpen-Adria-Universität Klagenfurt, Supported by FWF – Austrian Science Fund

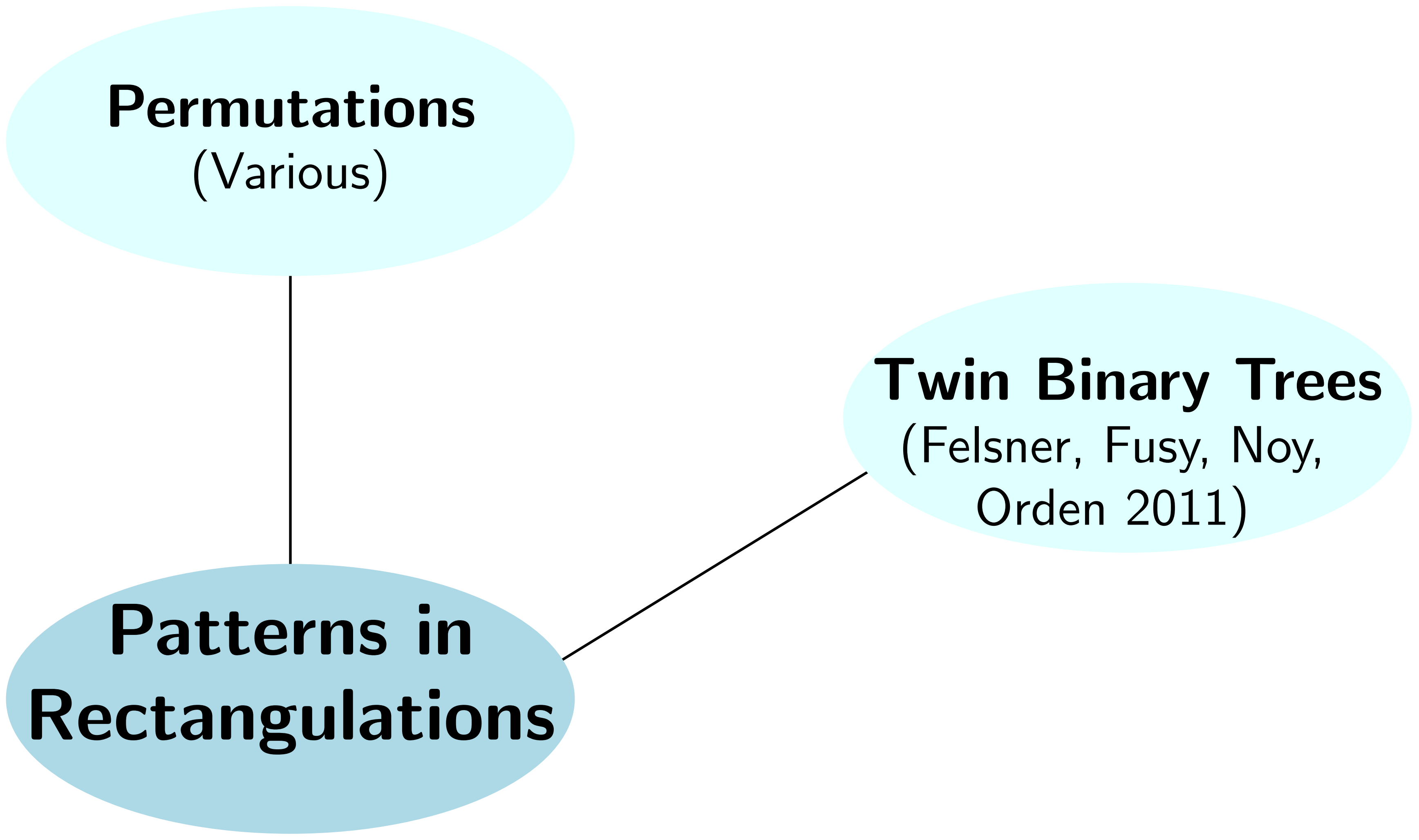
[3] University of Birmingham

[4] University of Kassel

Patterns in Rectangulations

Permutations
(Various)

Patterns in Rectangulations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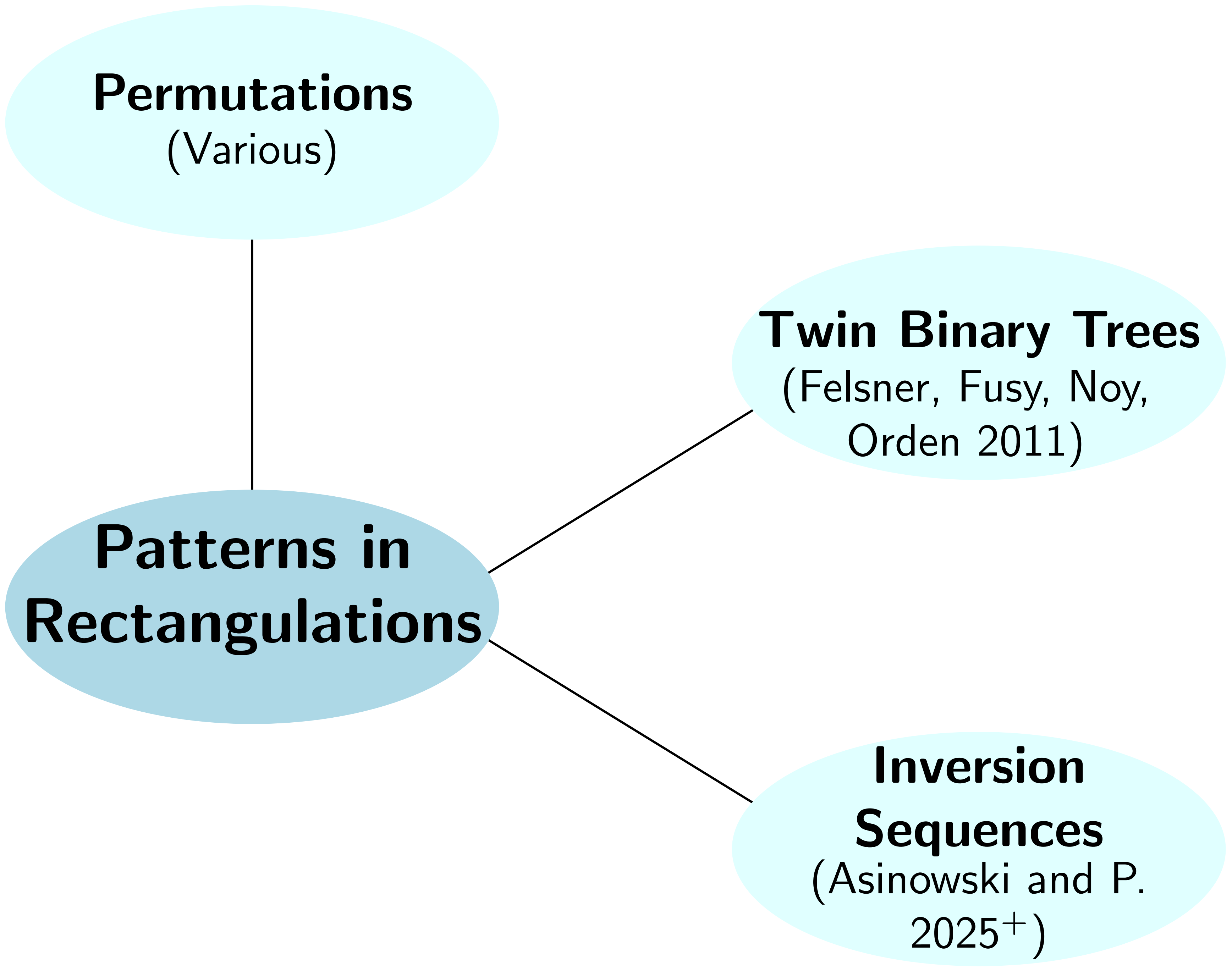
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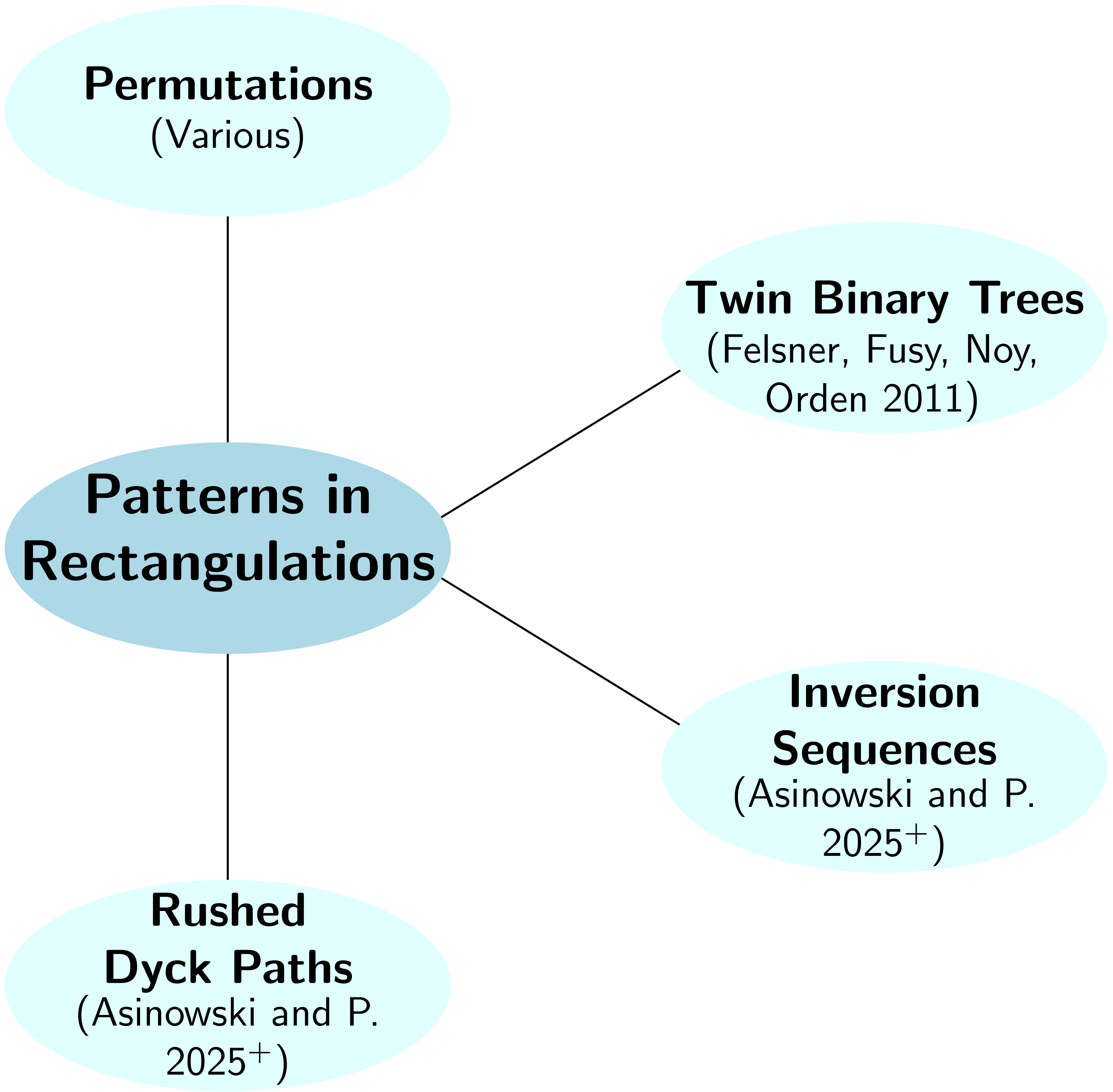
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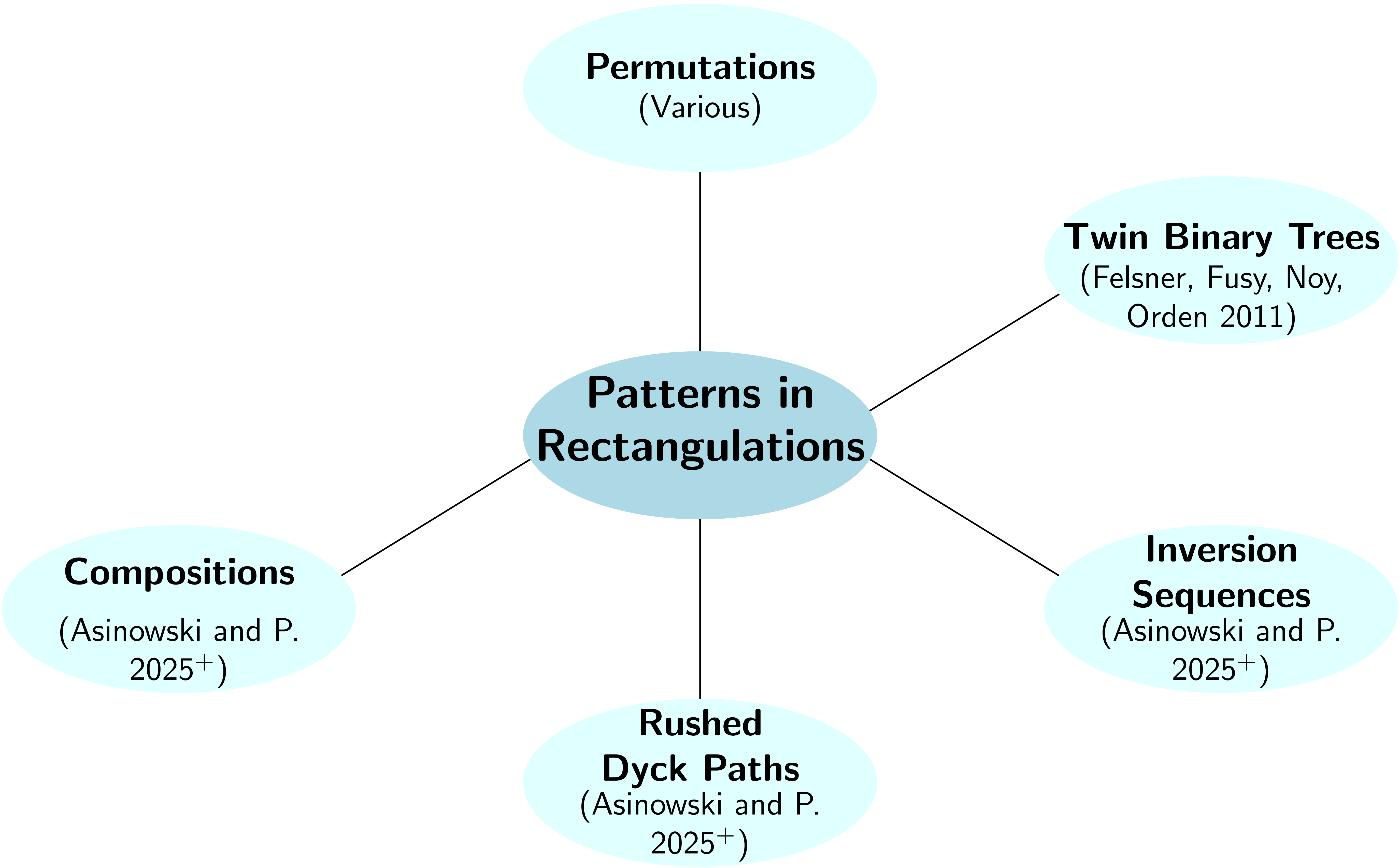
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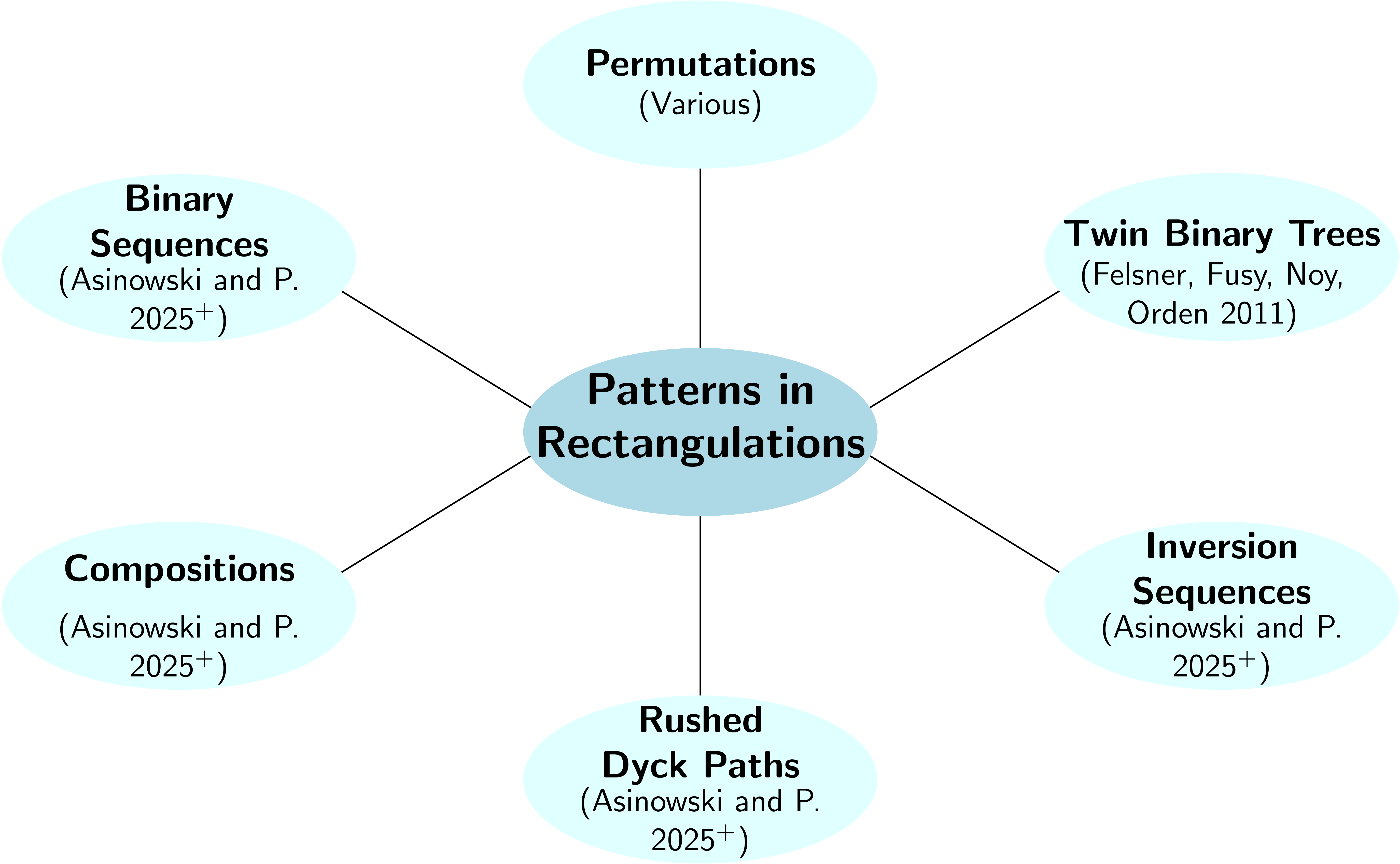
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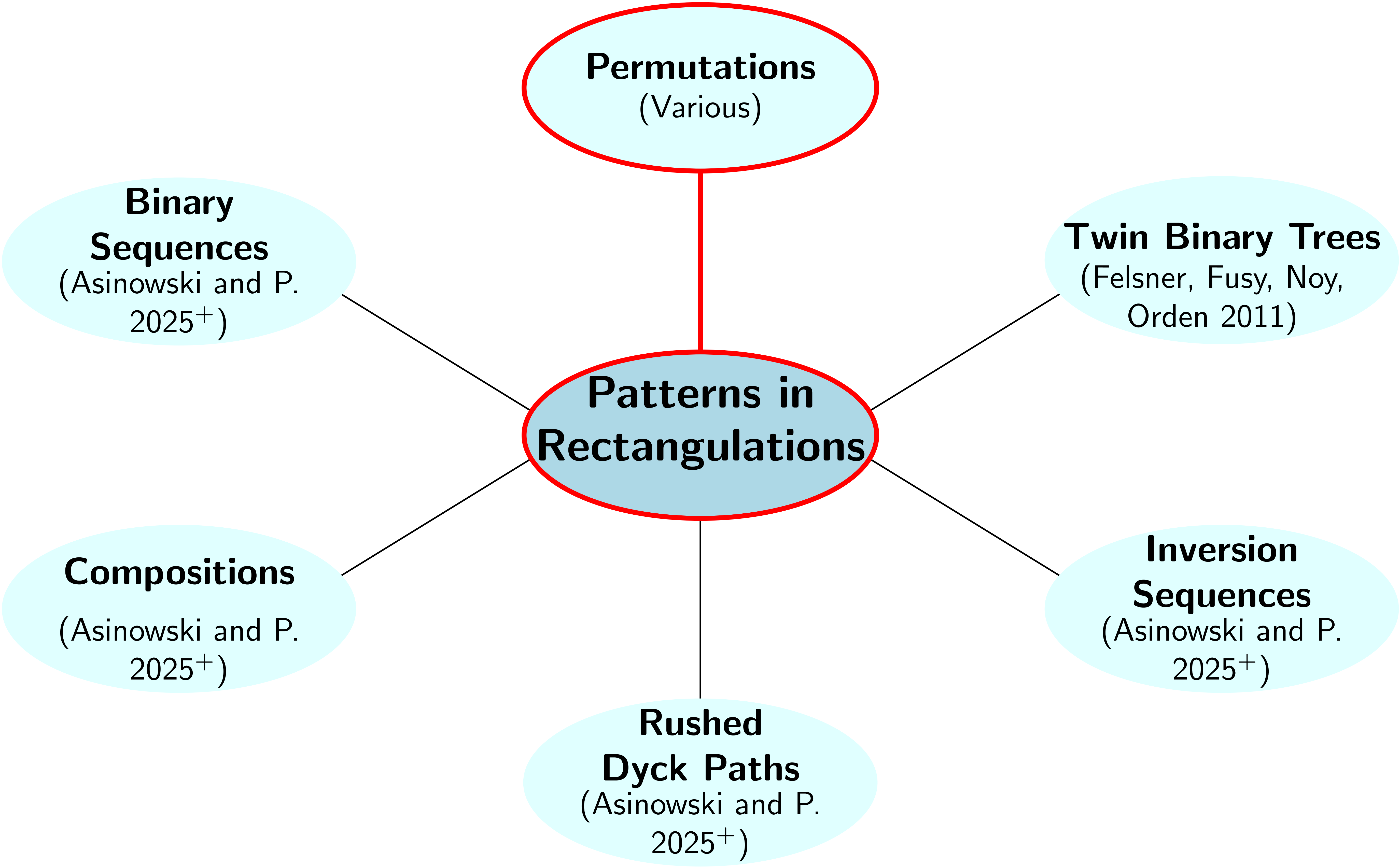
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Definition

A *diagonal rectangulation* is a rectangulation in which each rectangle meets the northwest-southeast diagonal.

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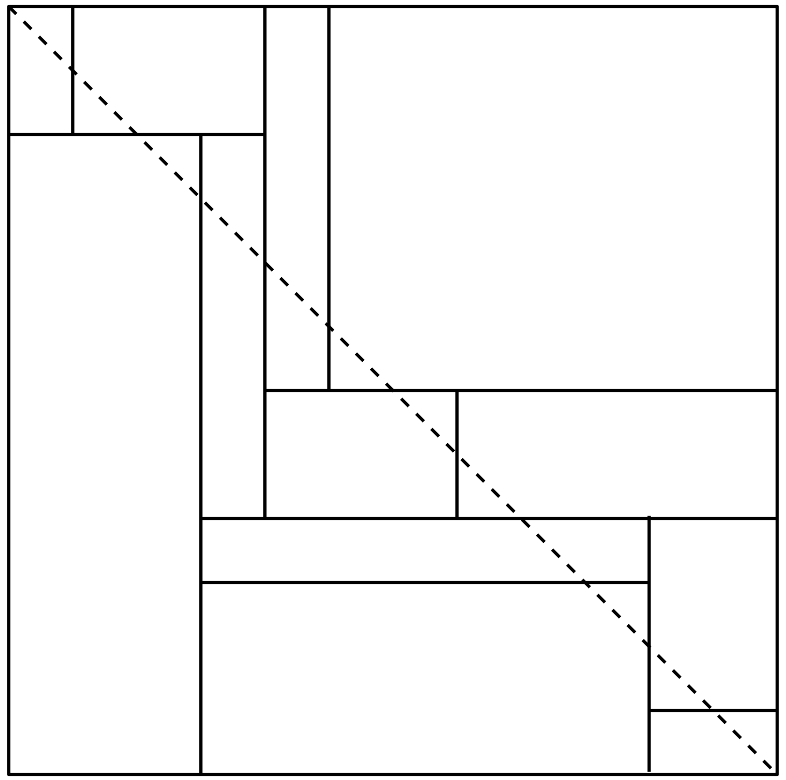

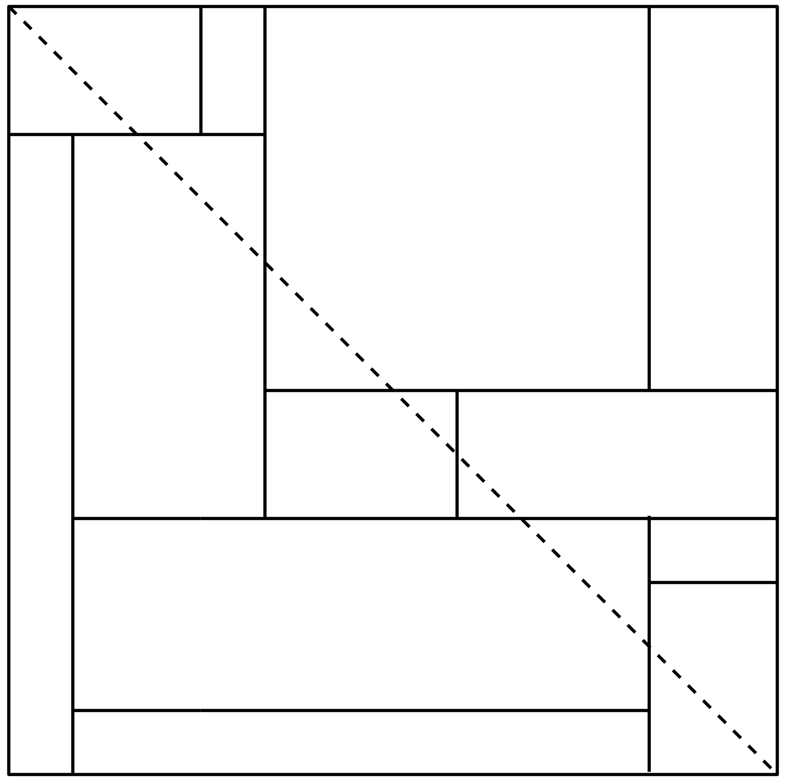

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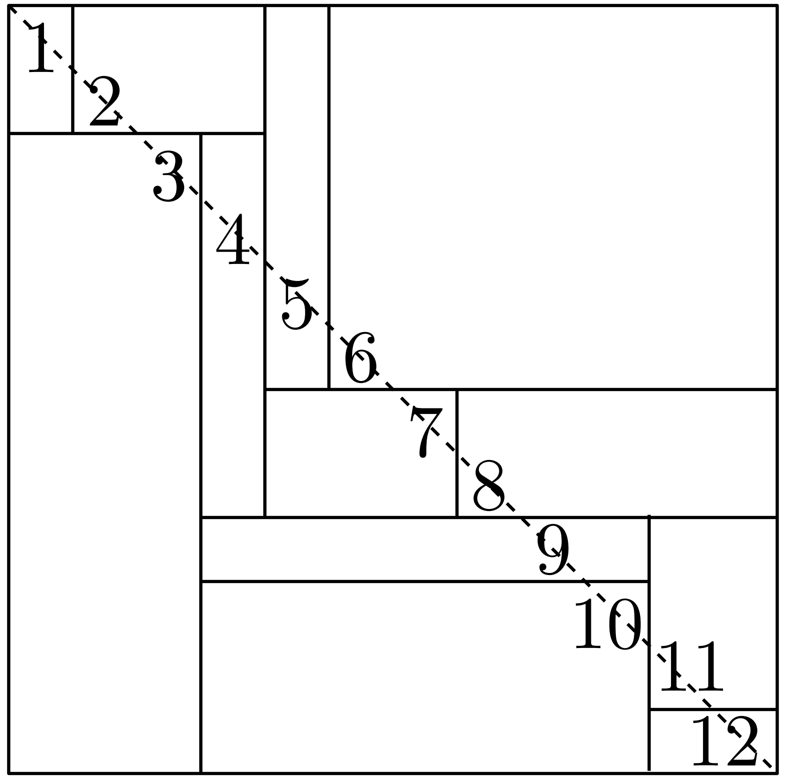

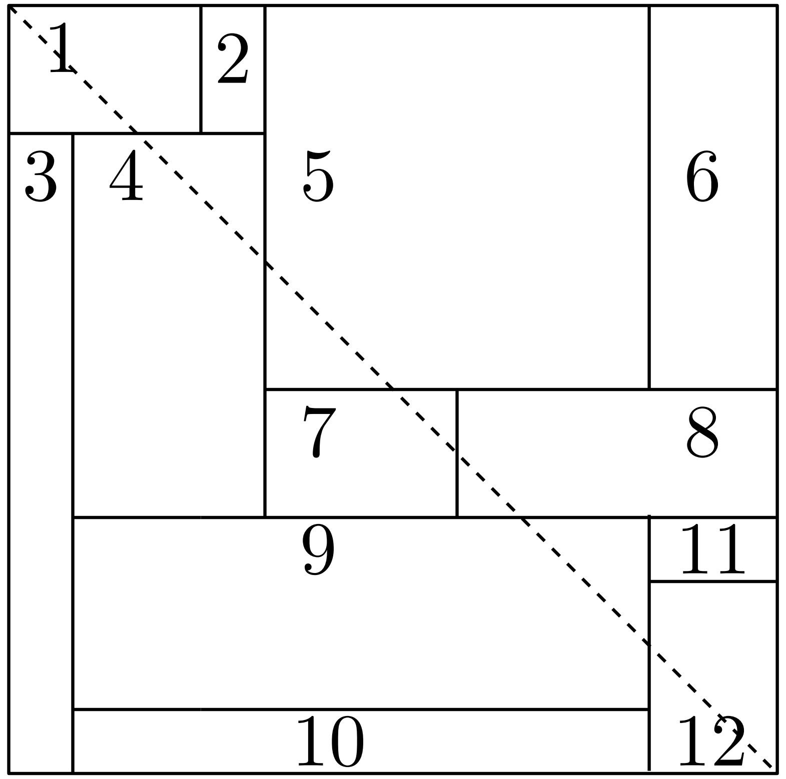

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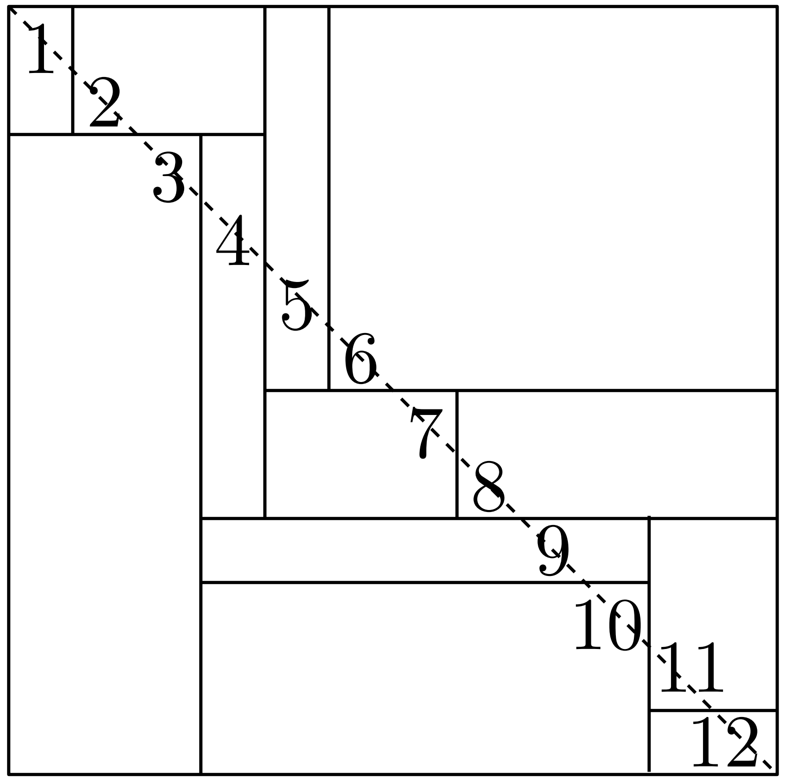

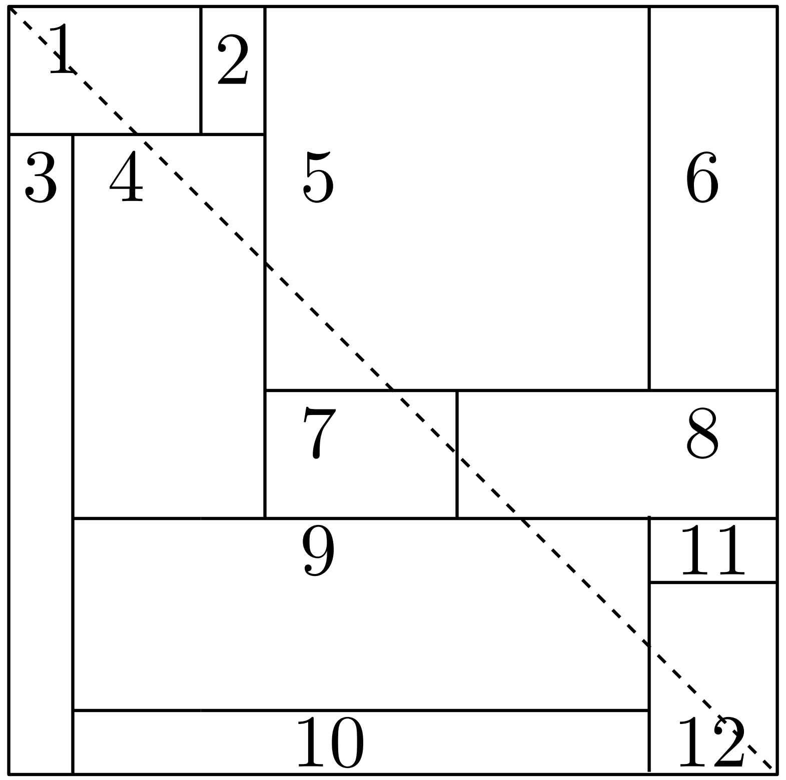

Proposition (Reading 2012)

Every weak rectangulation has a unique representative which is a diagonal rectangulation.

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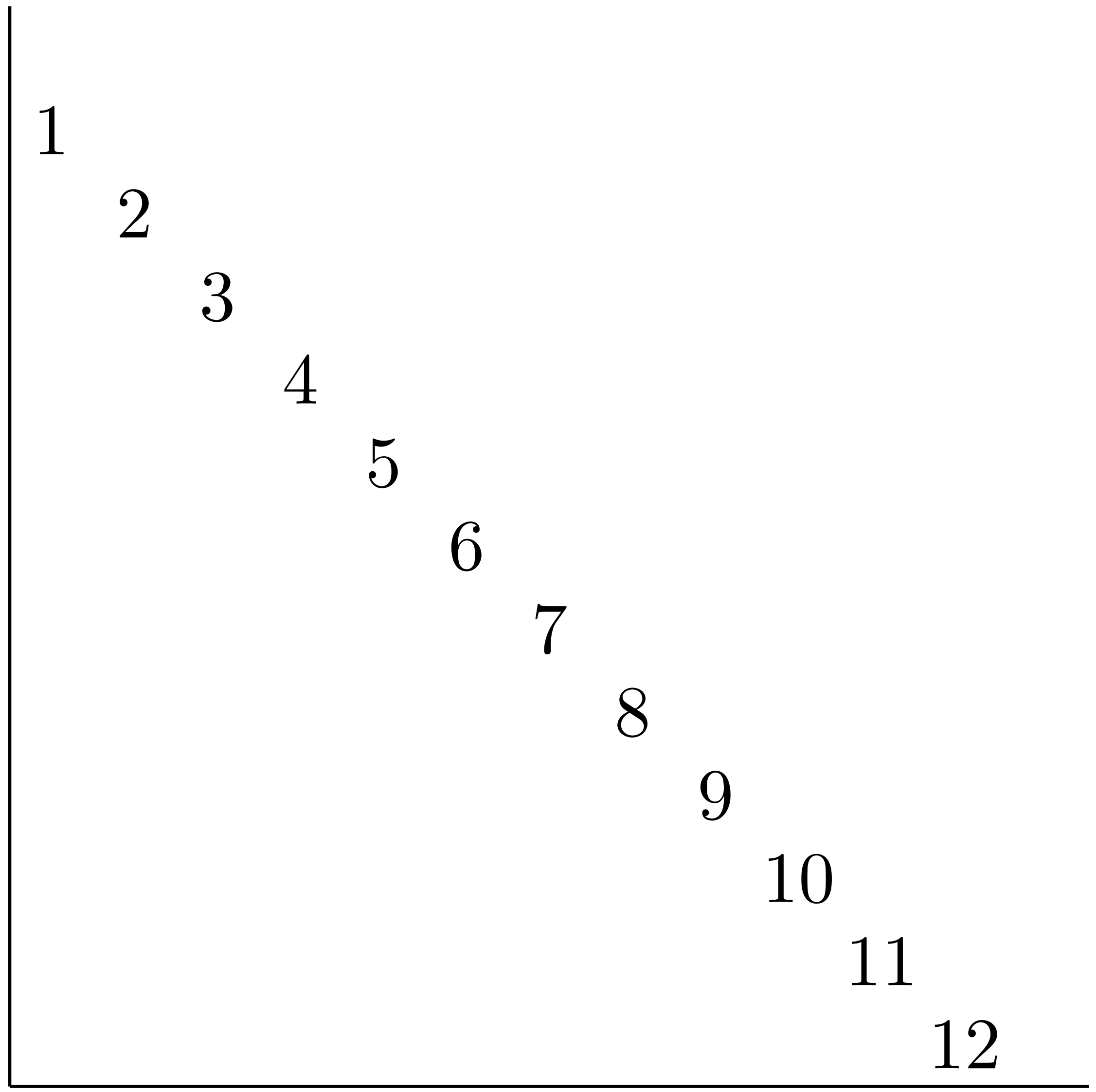

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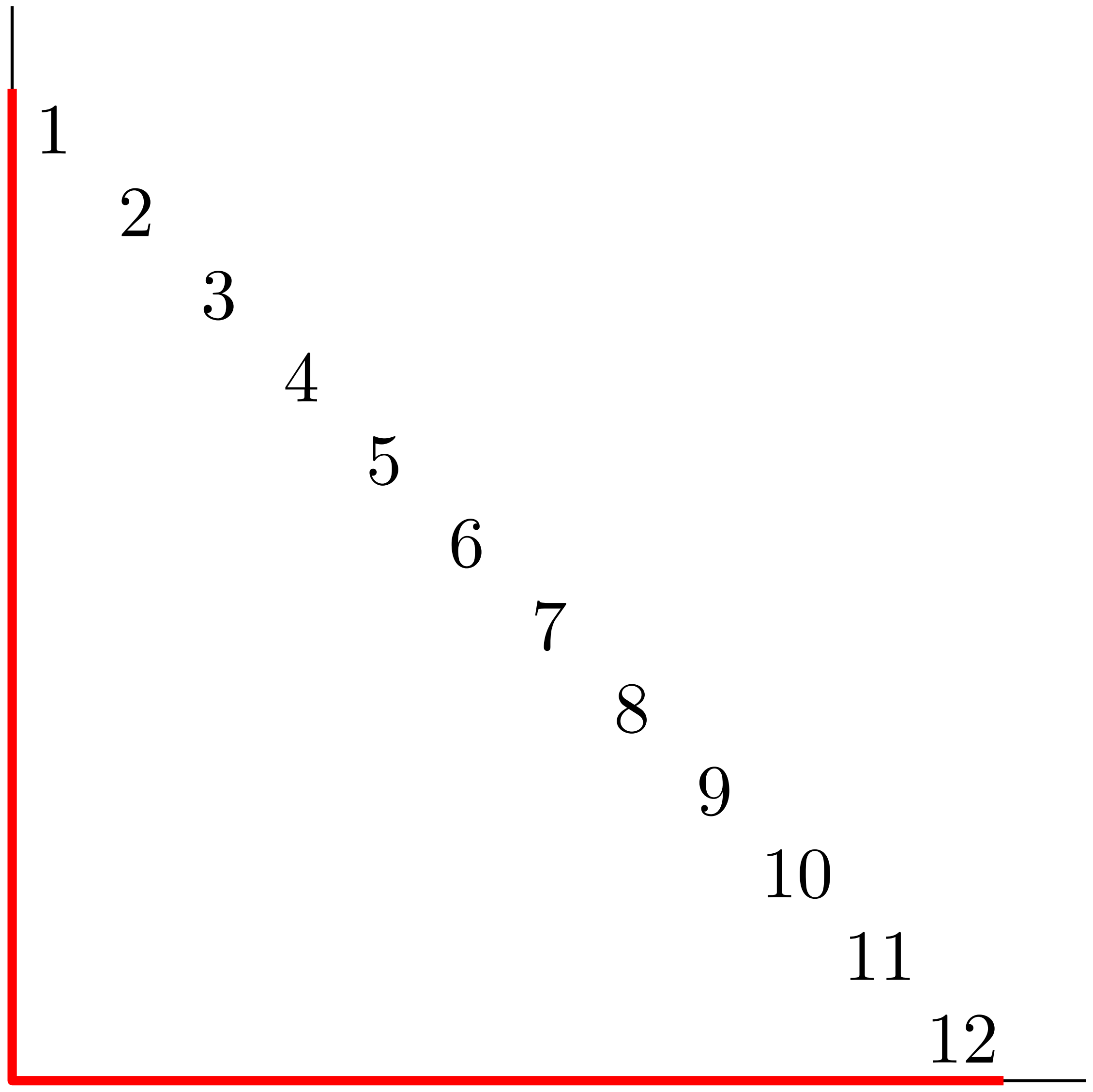

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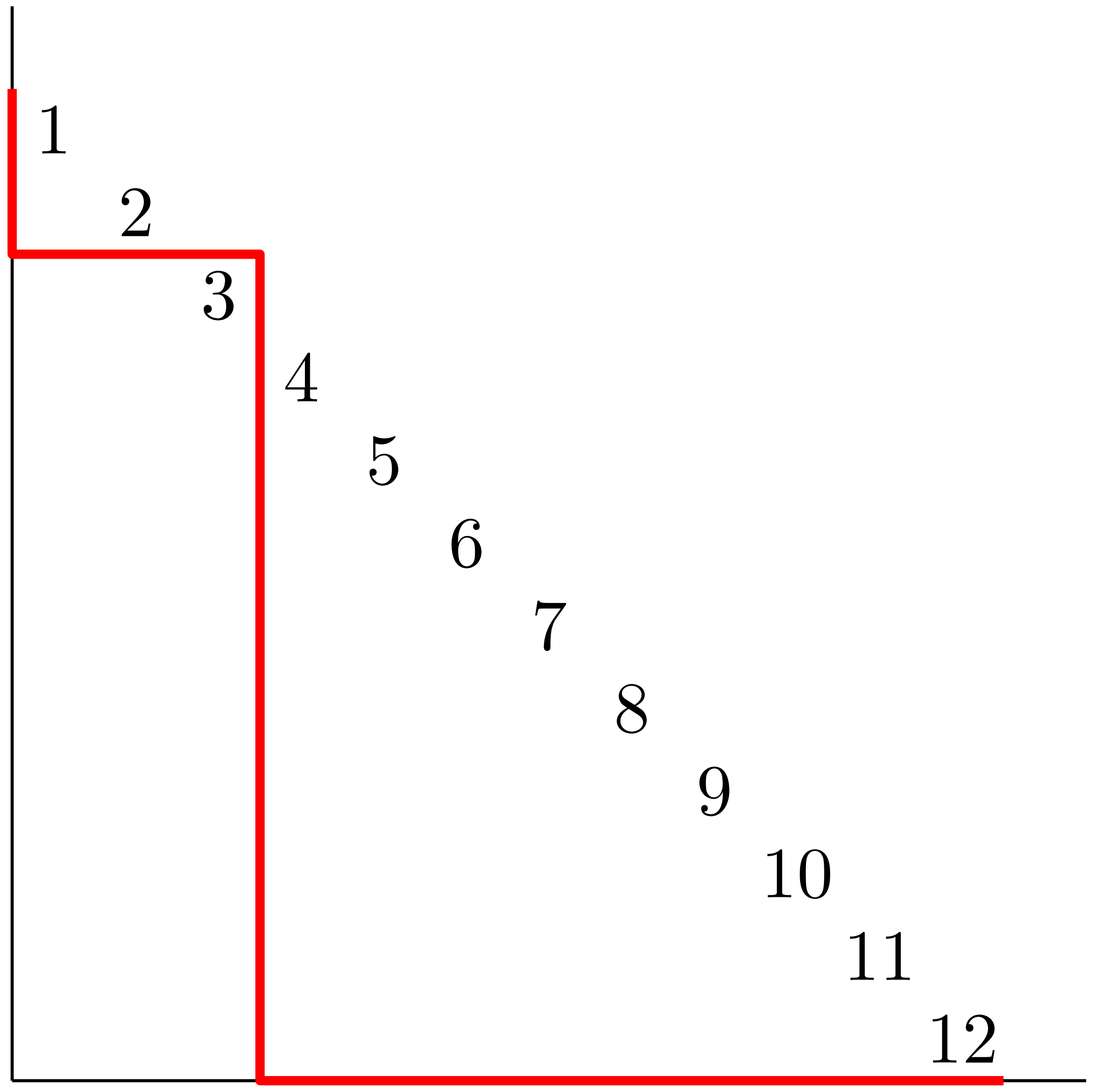

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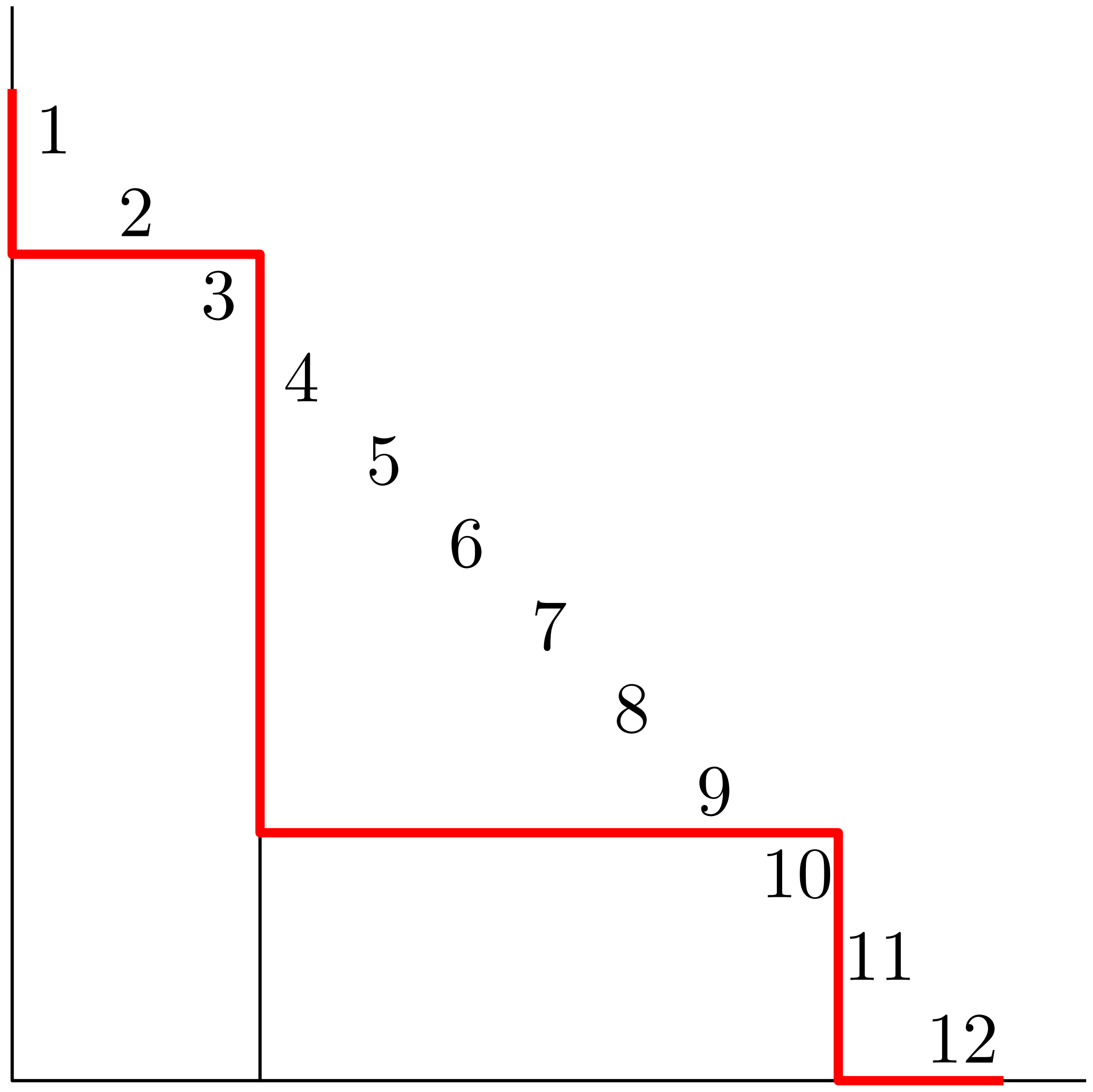

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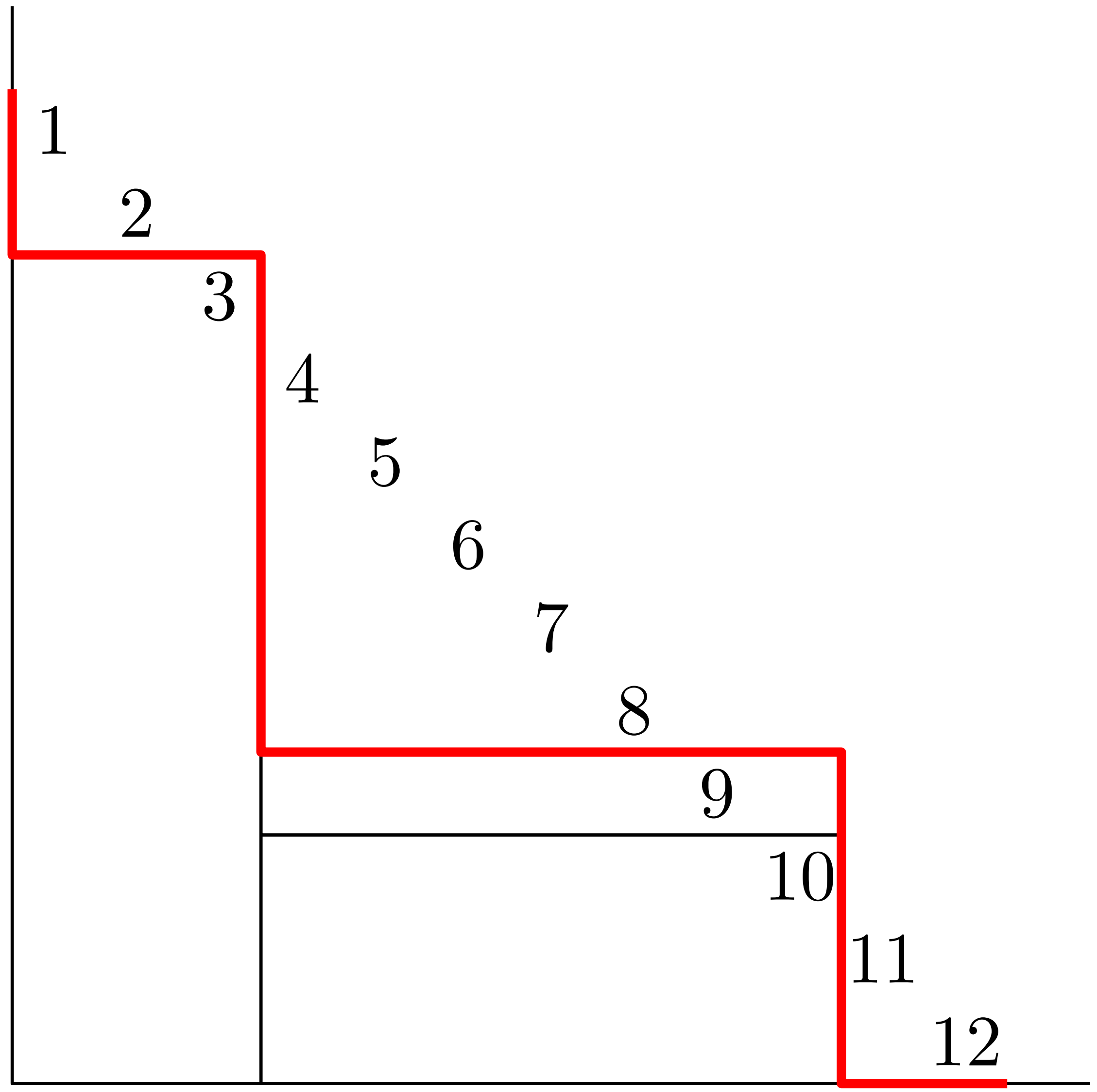

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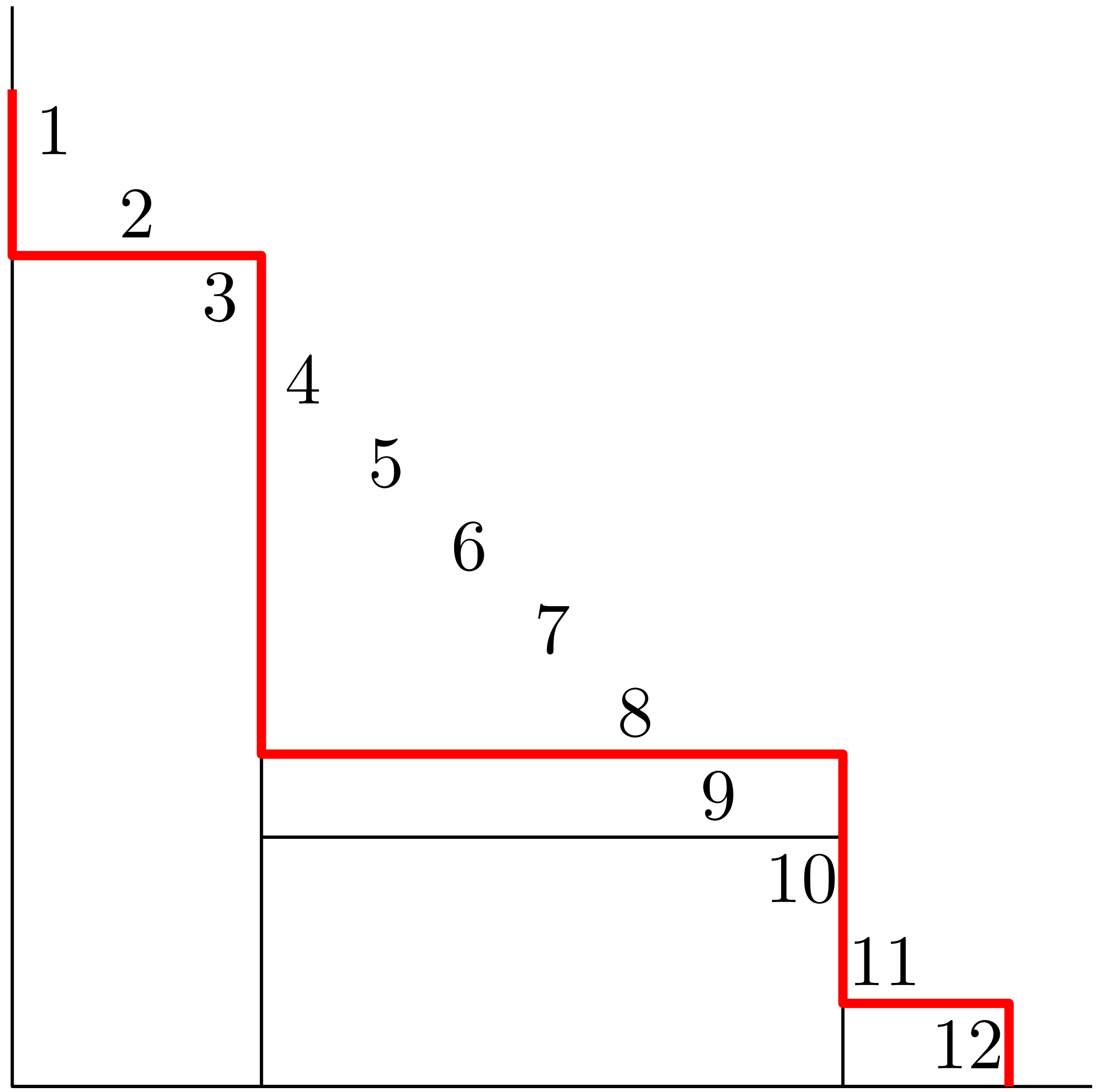

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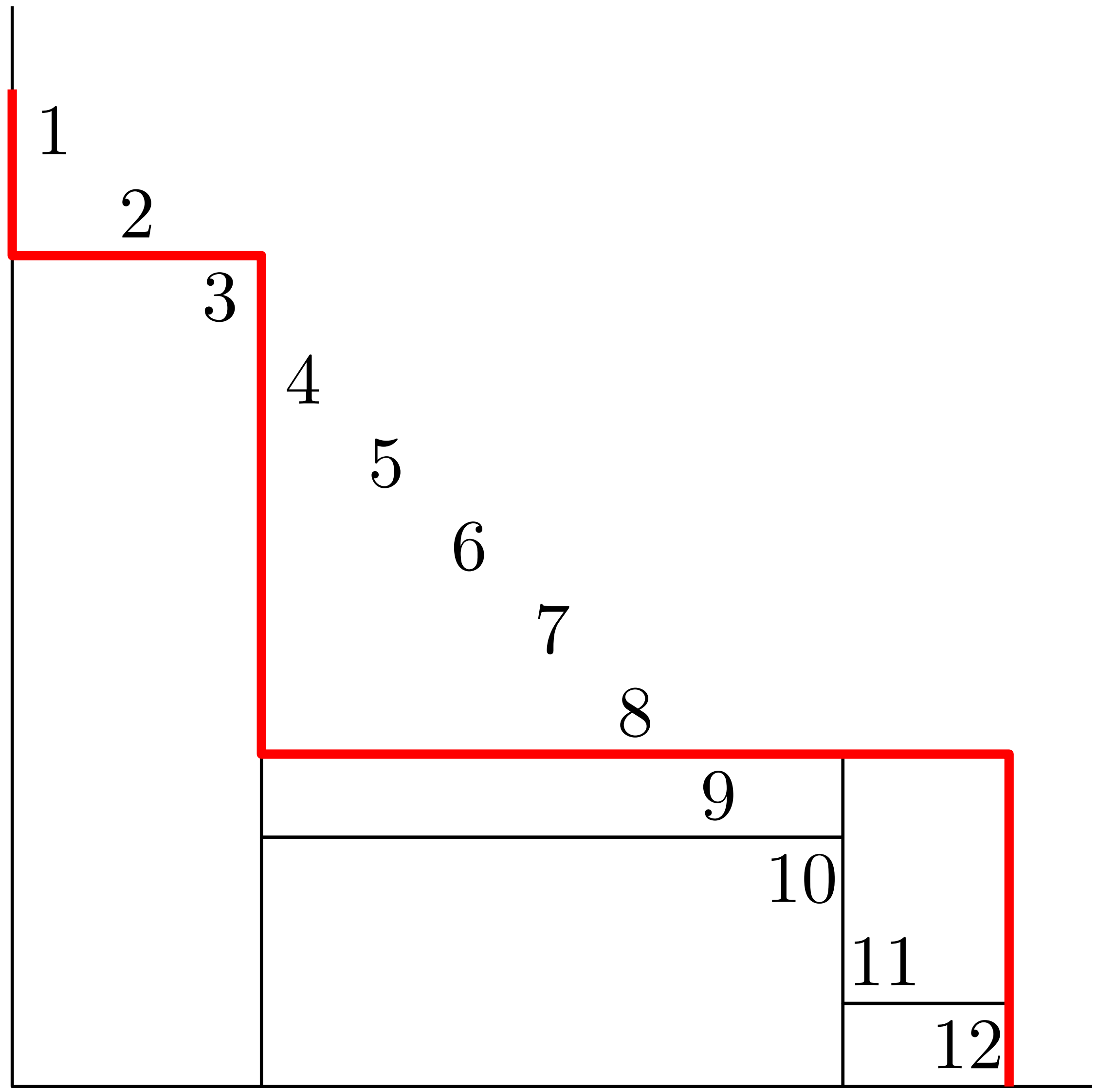

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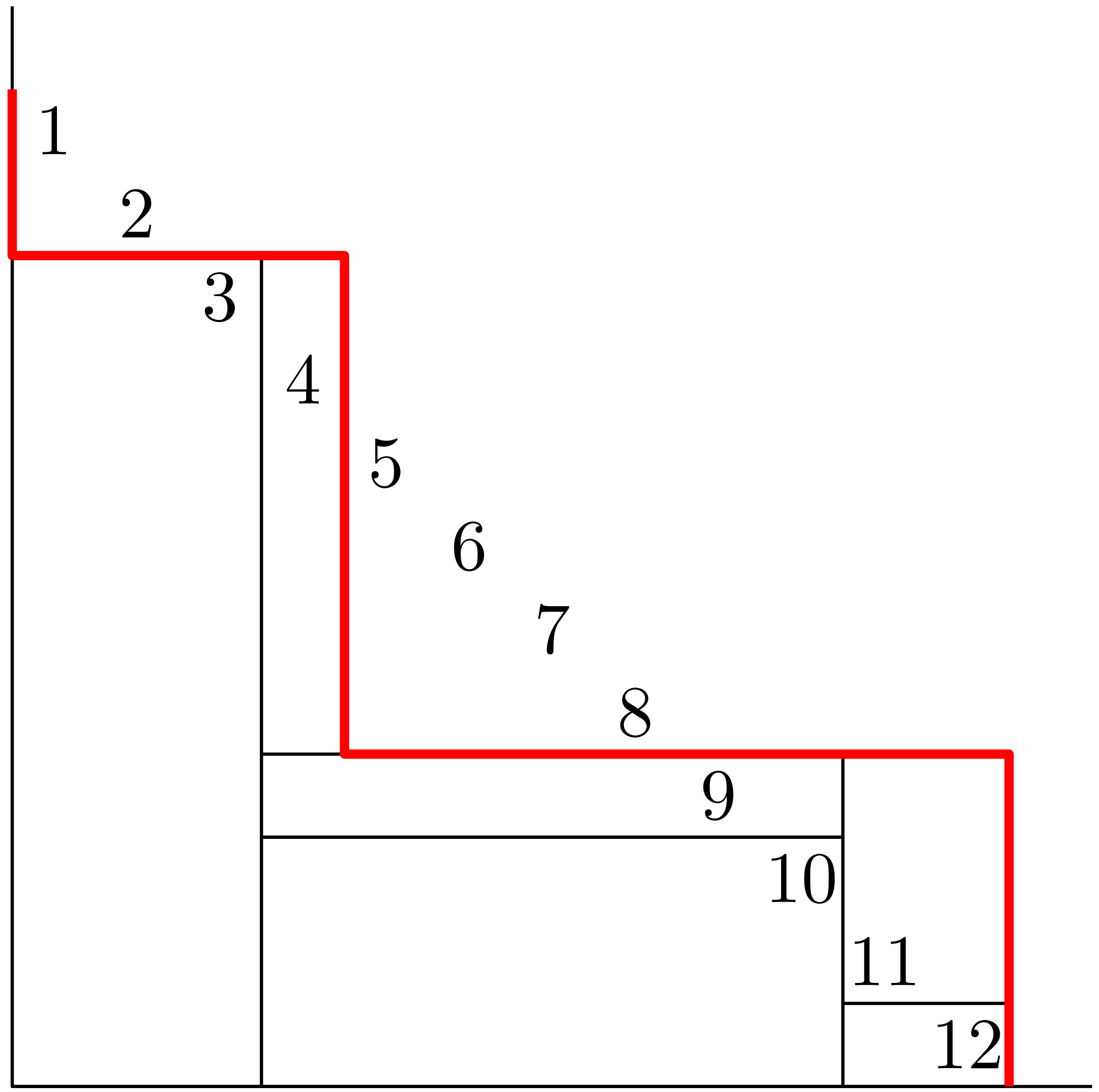

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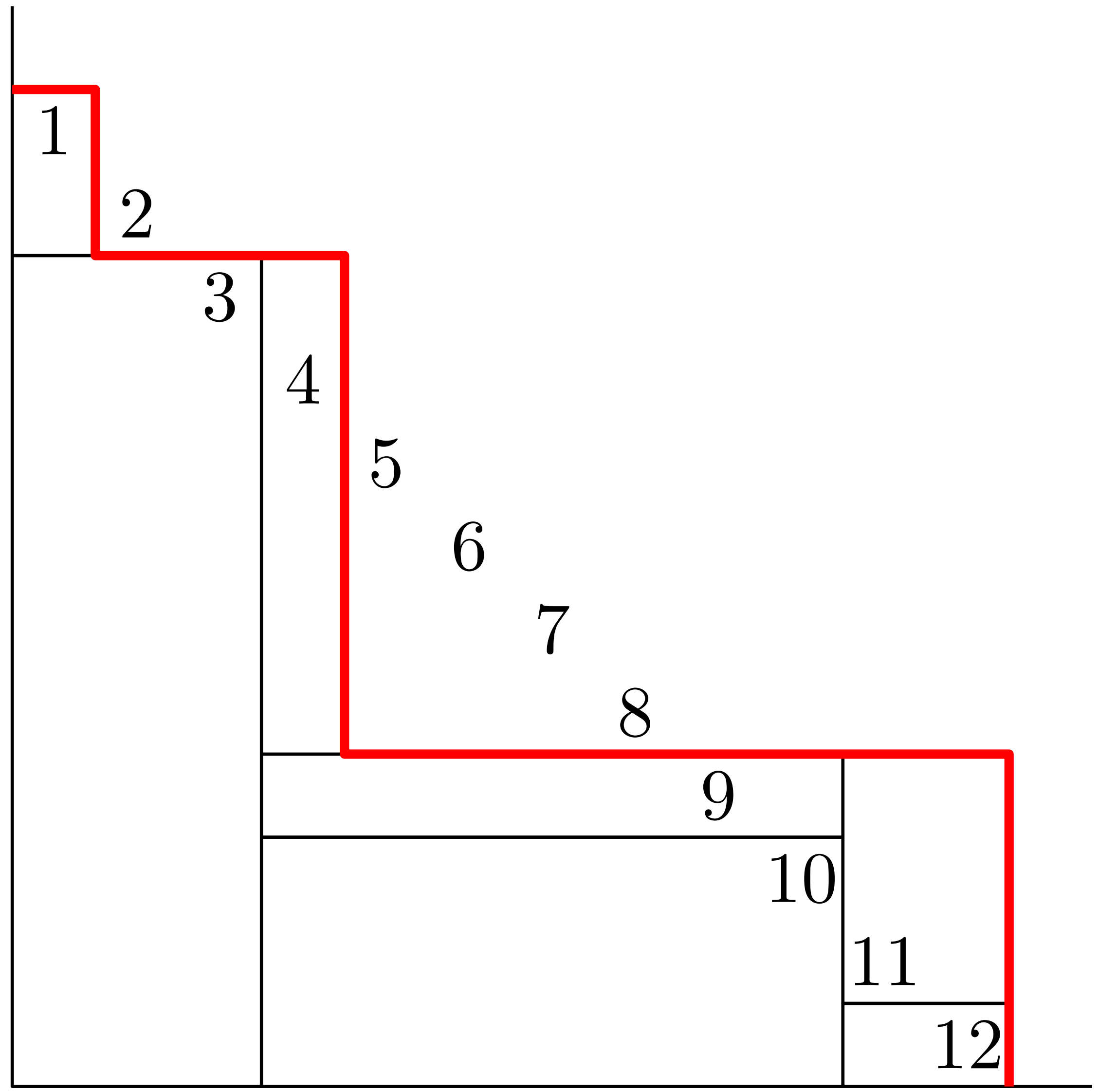

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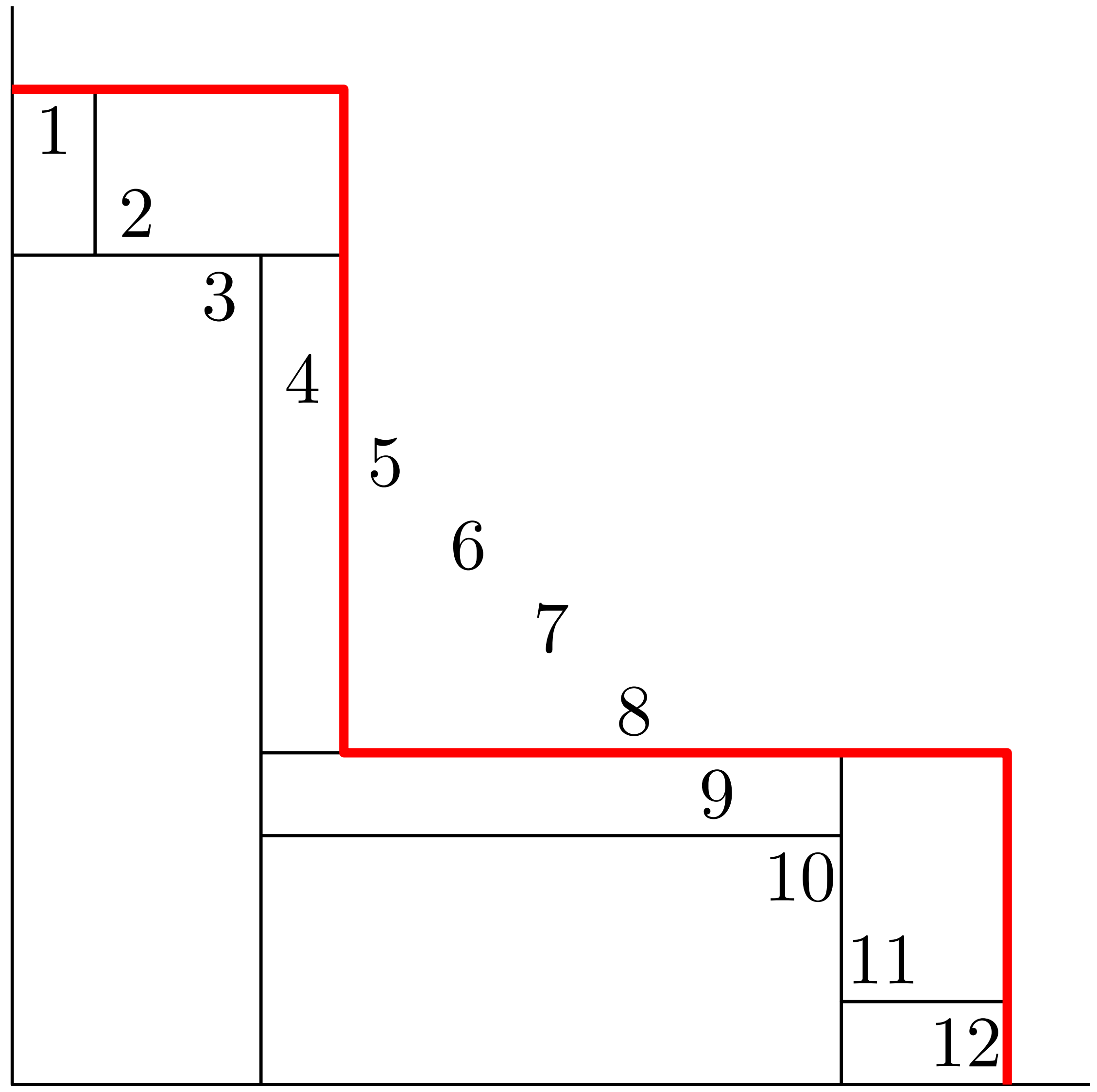

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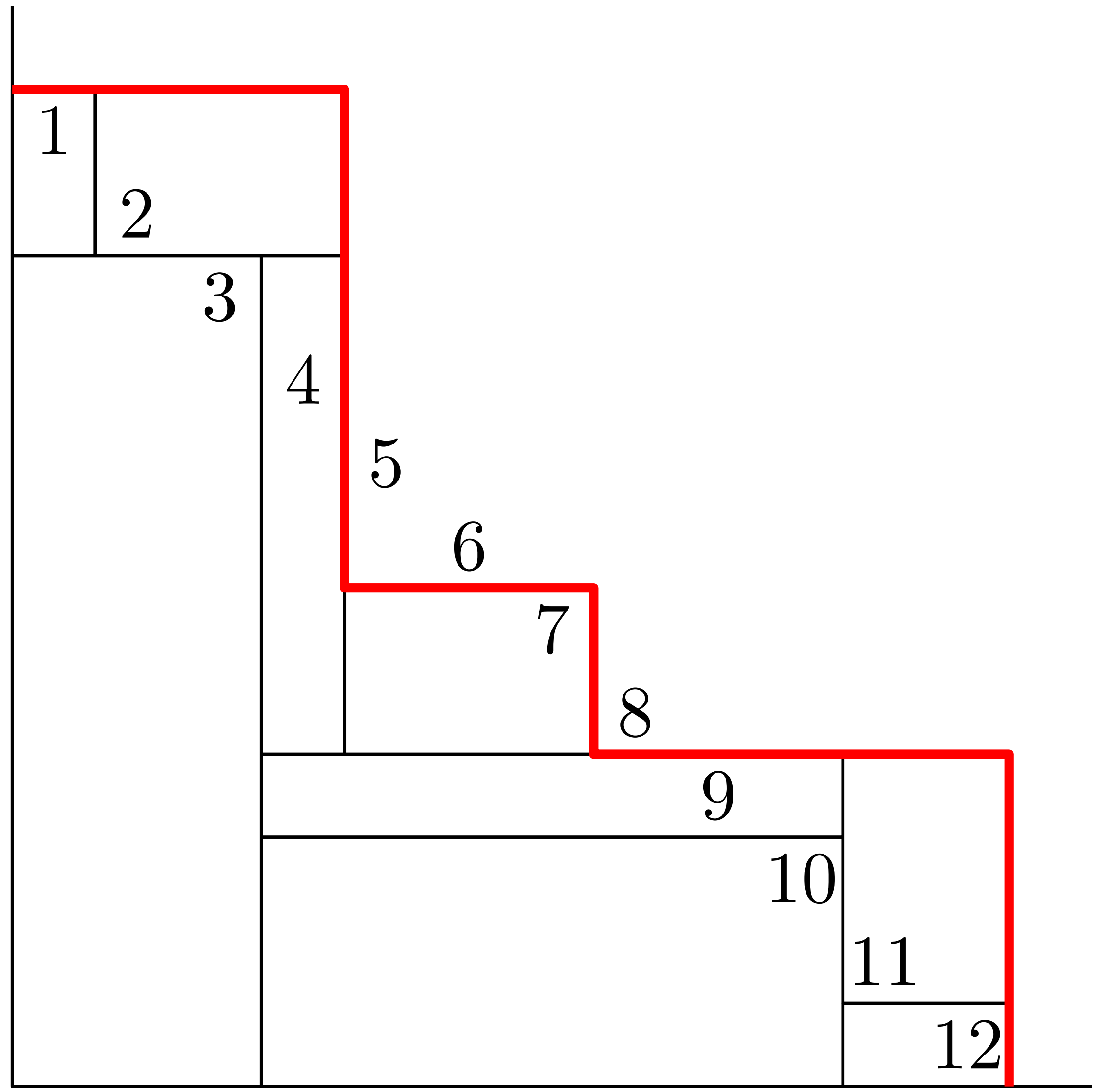

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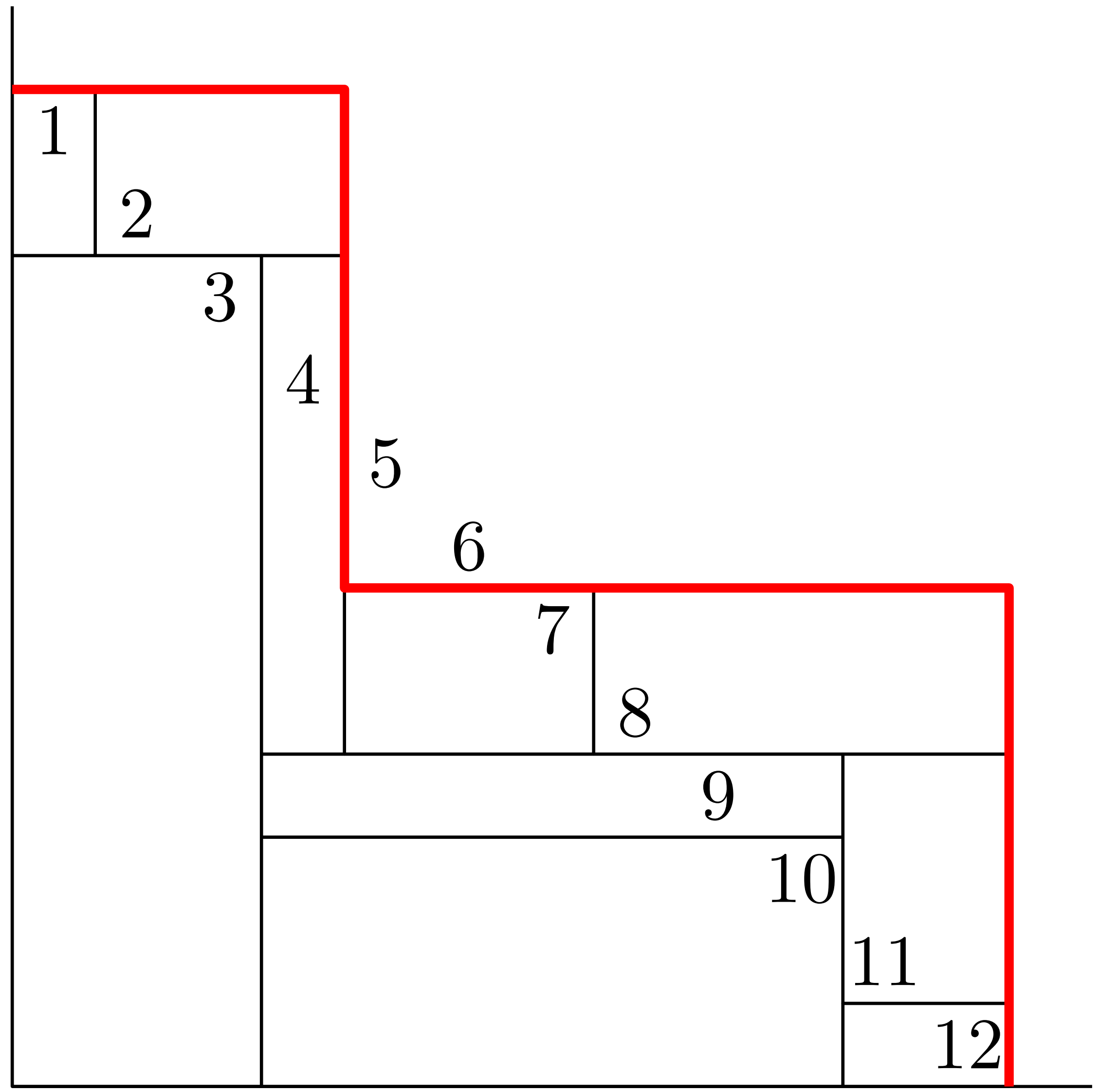

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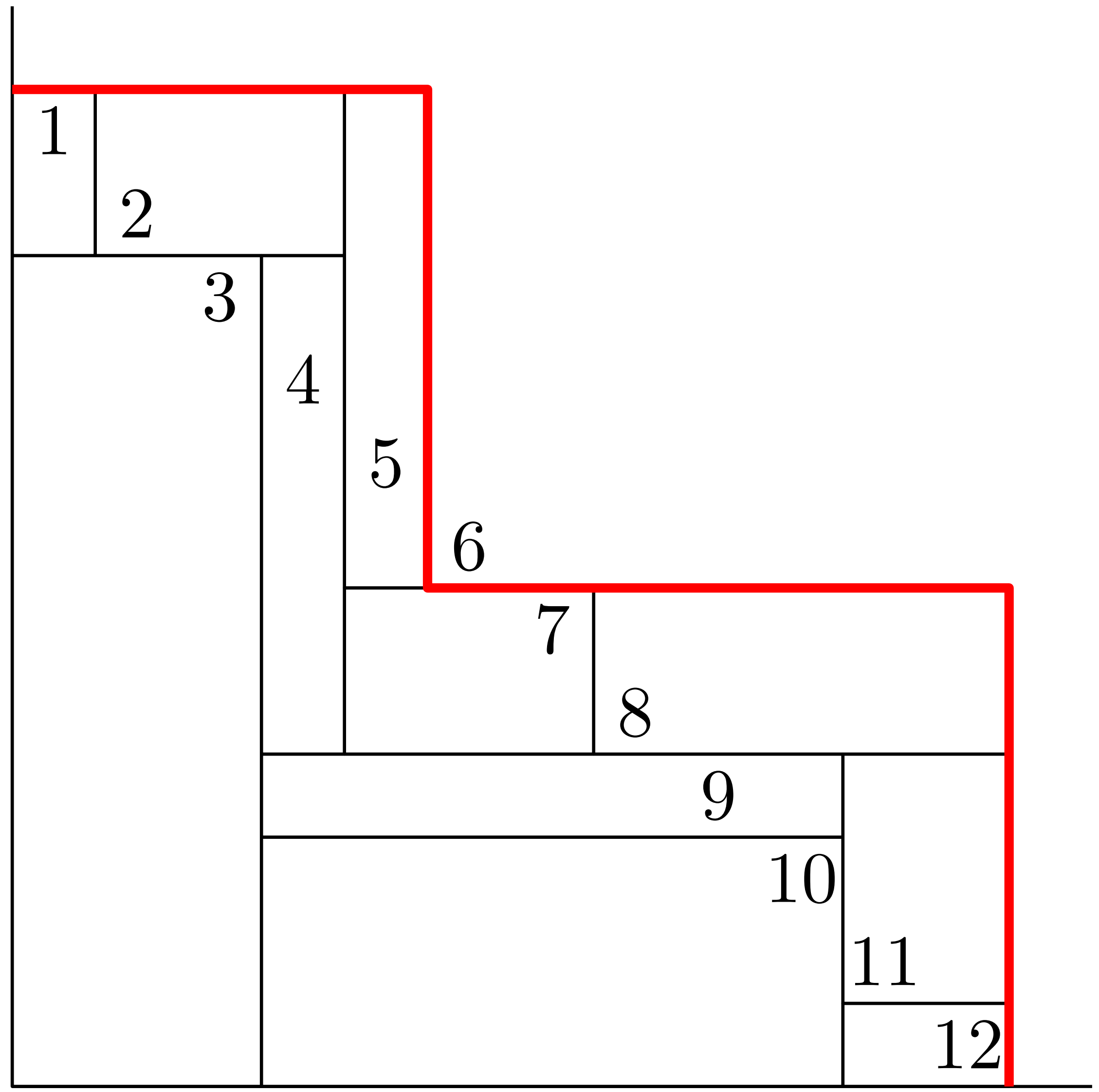

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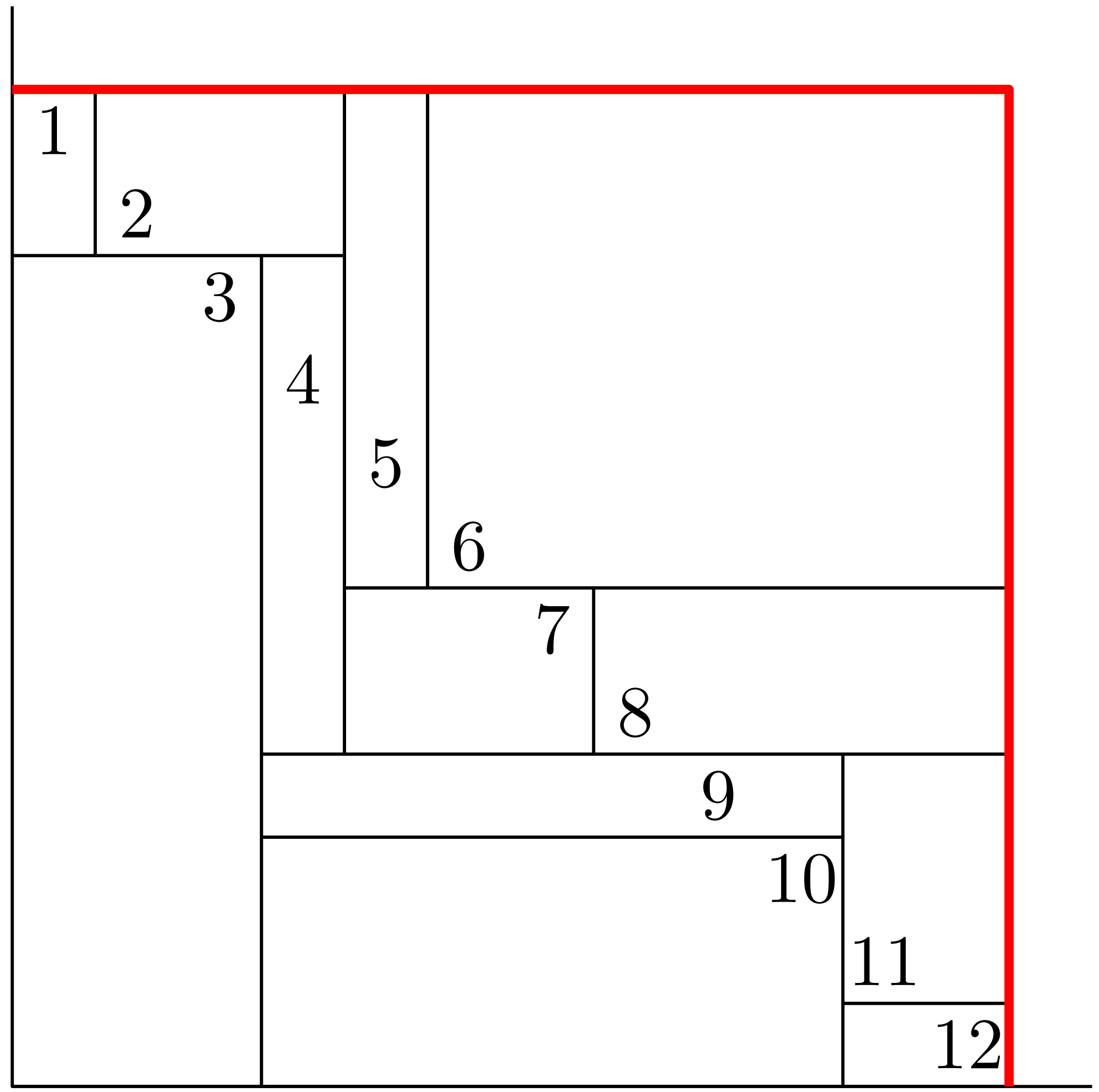

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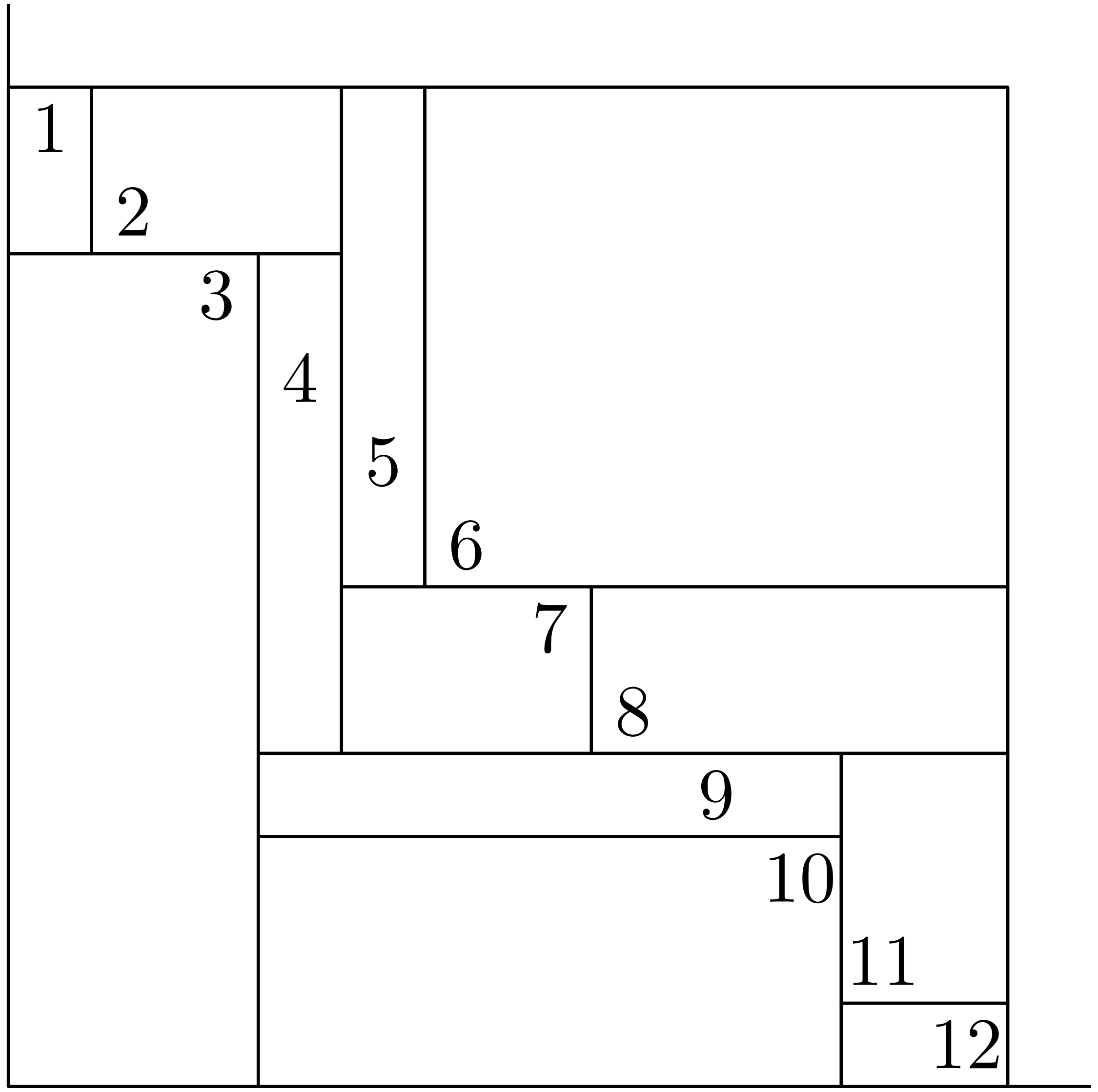

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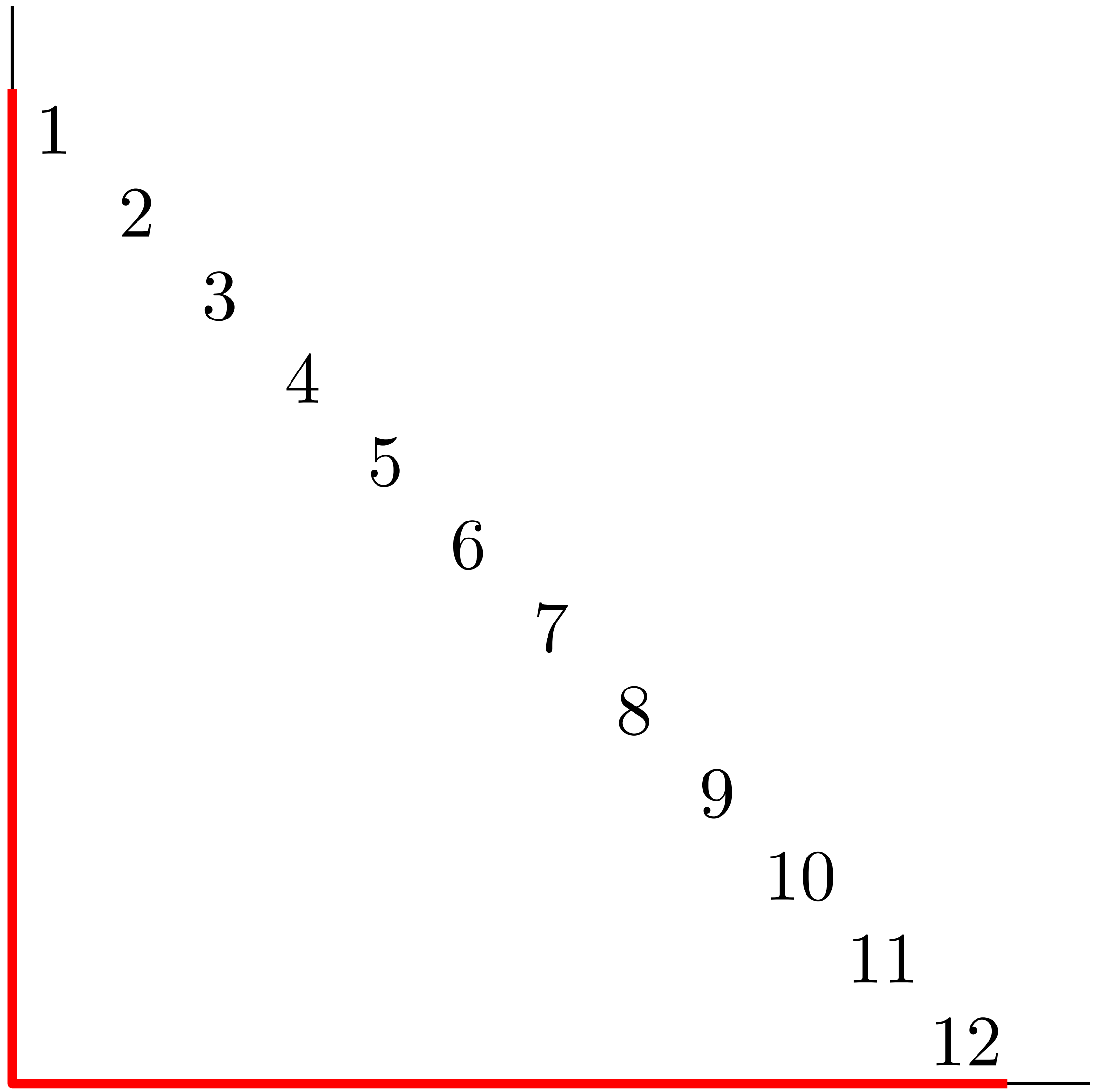

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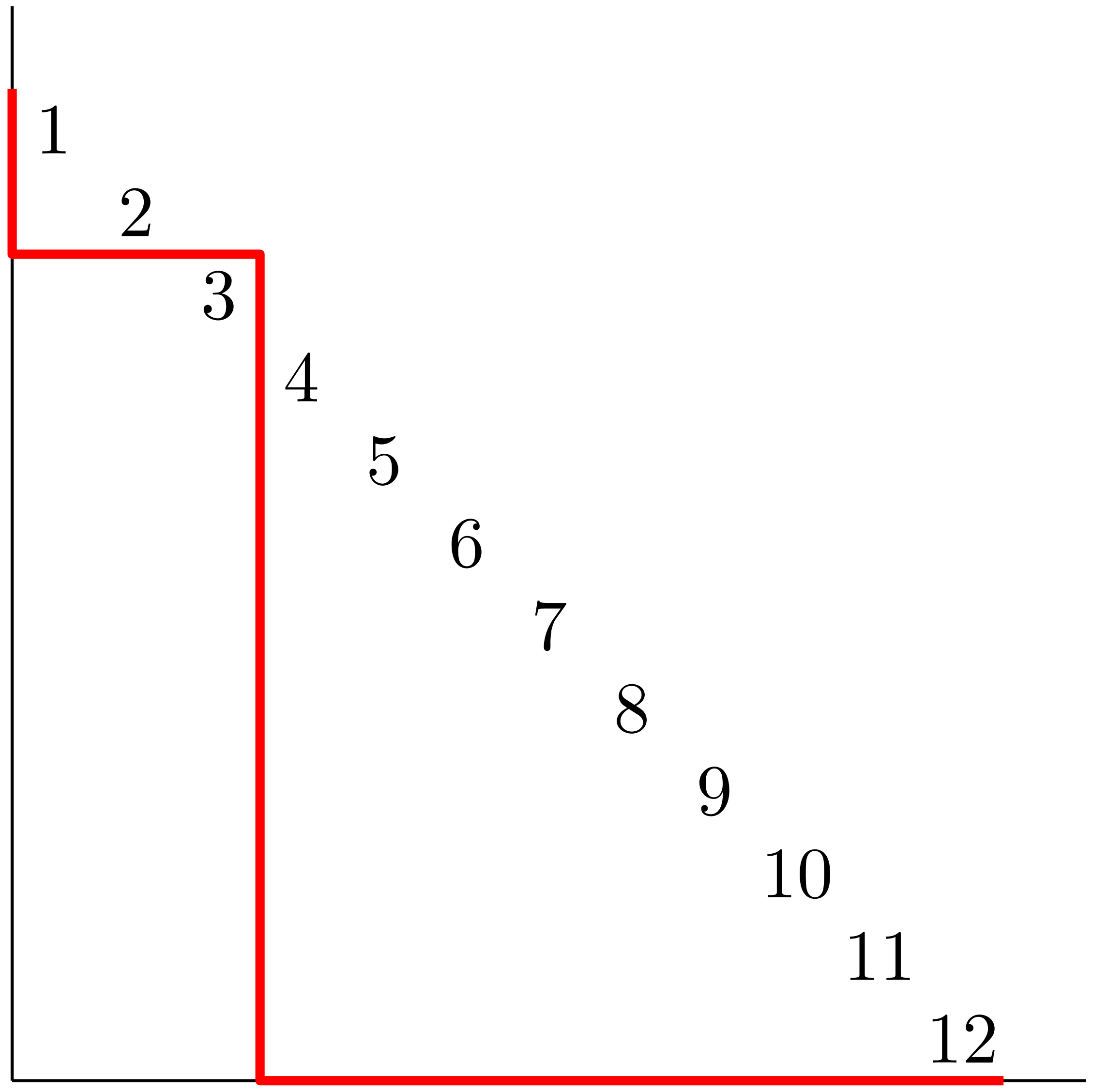

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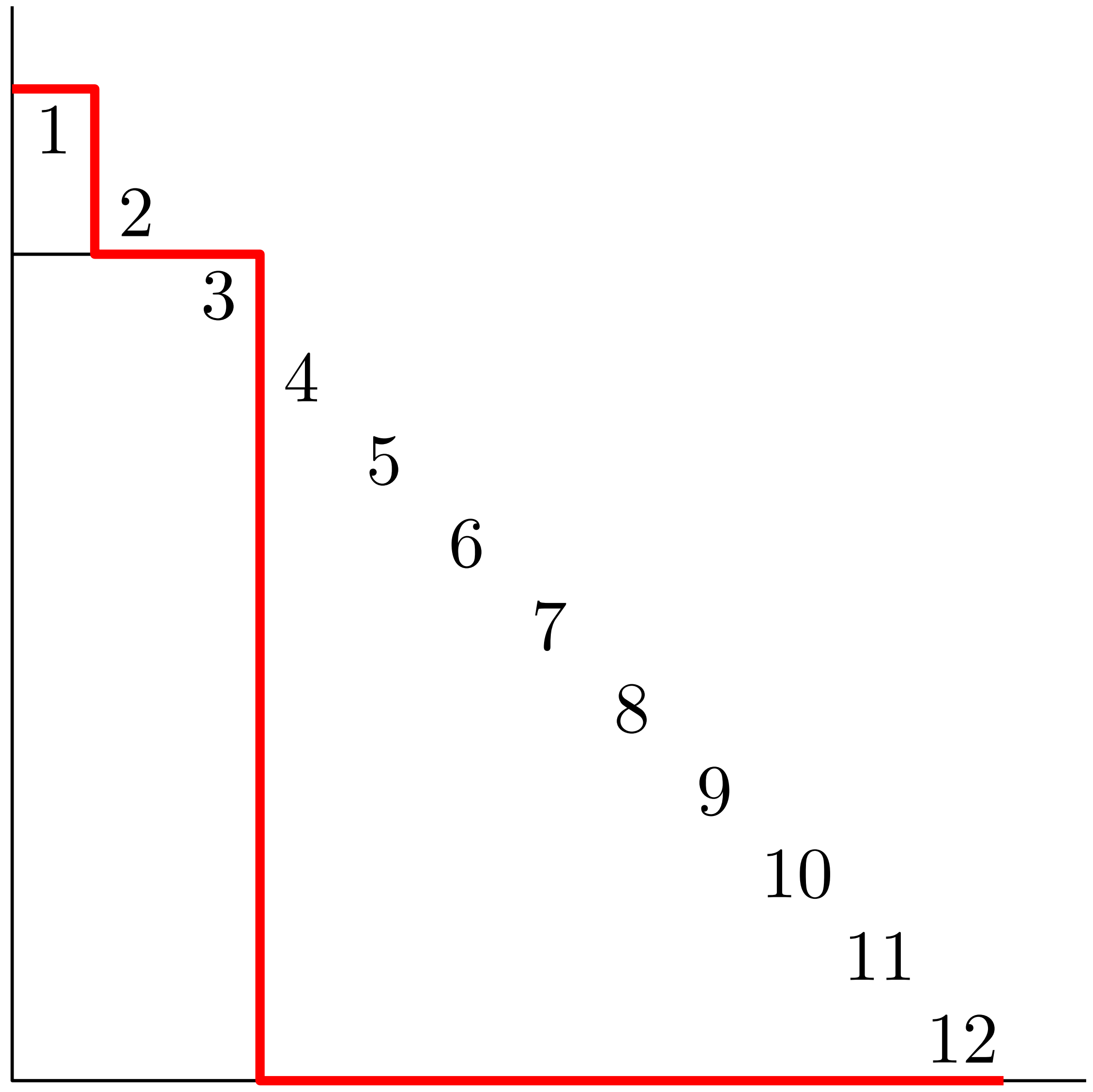

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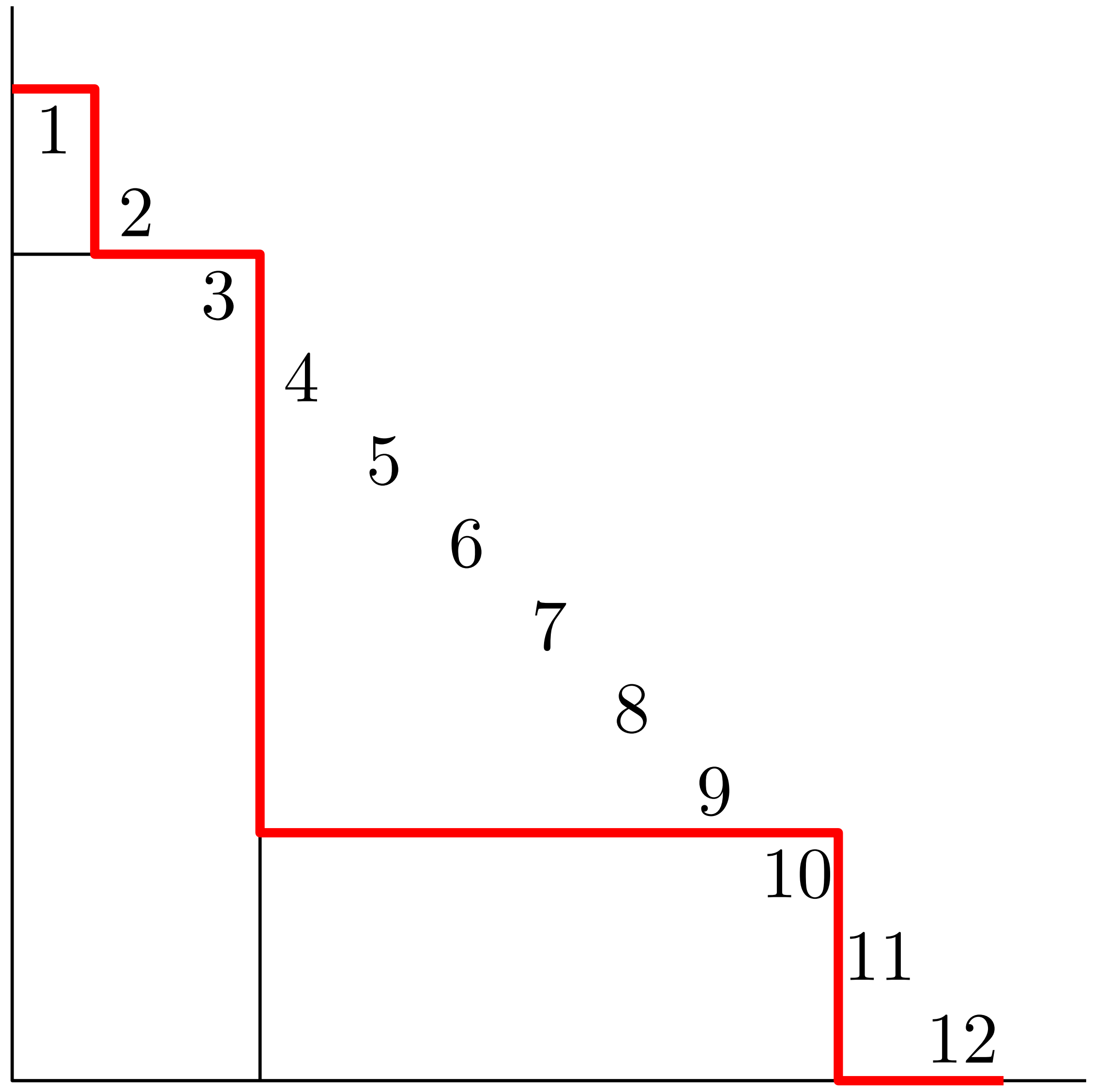

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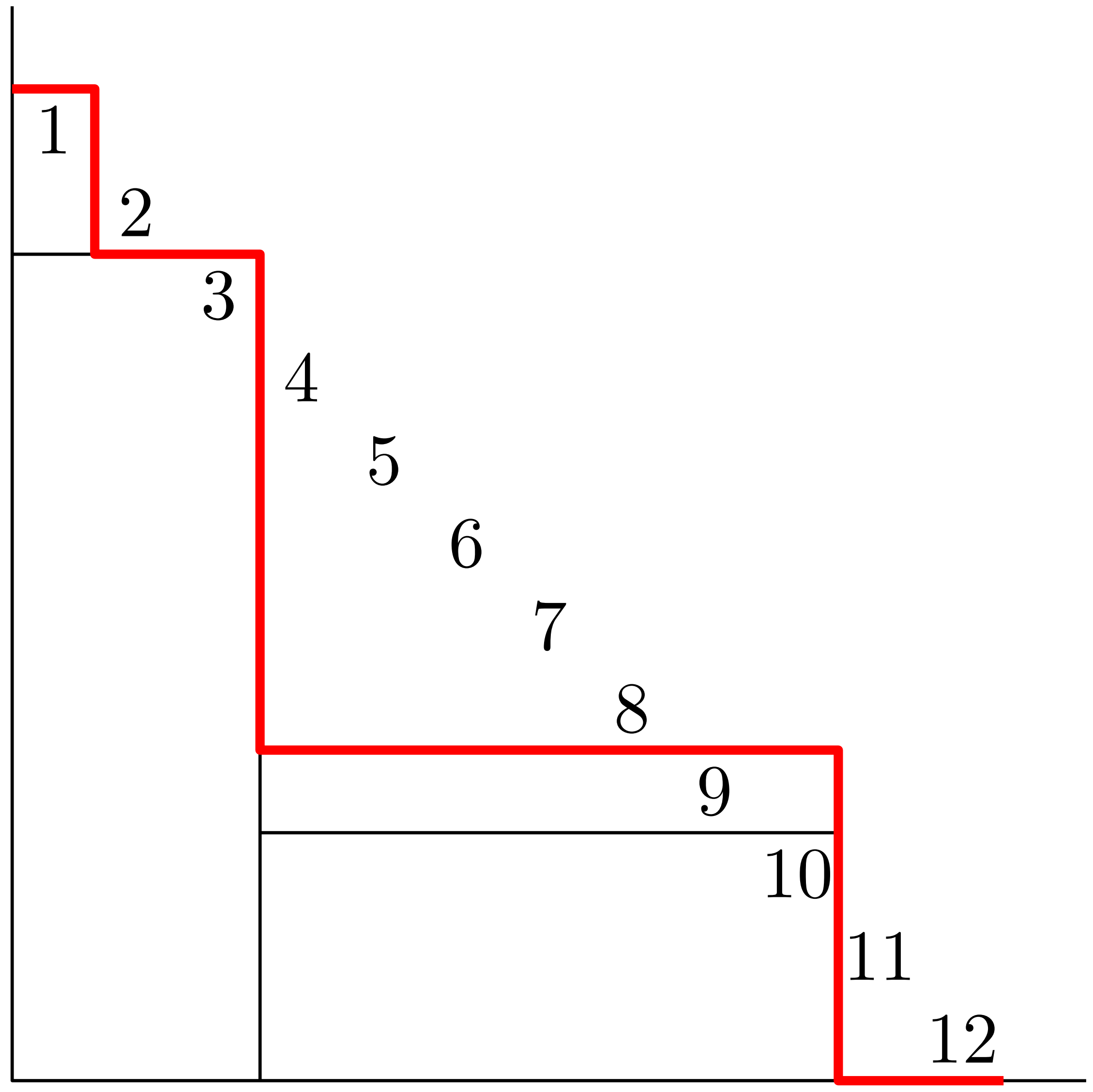

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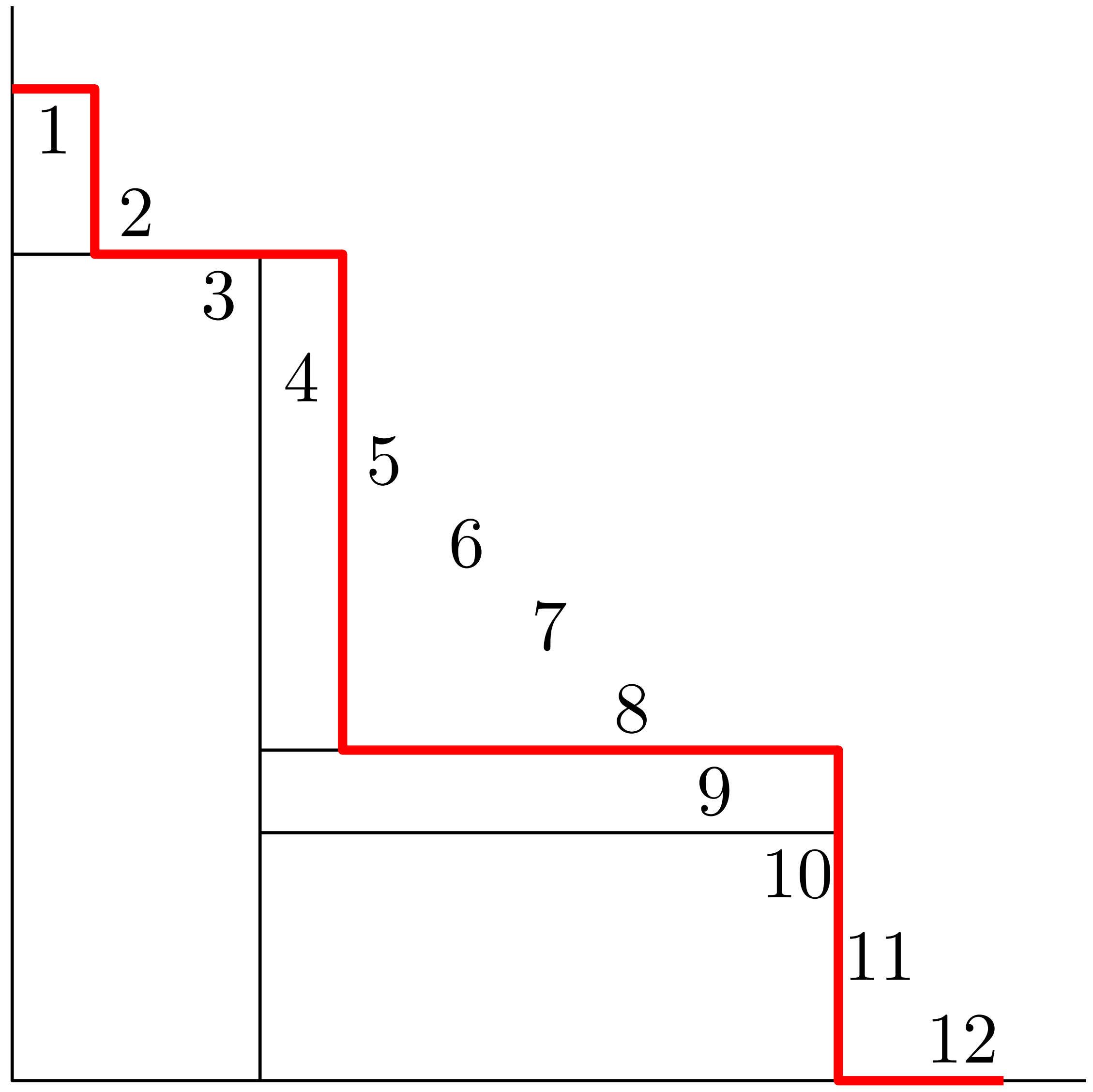

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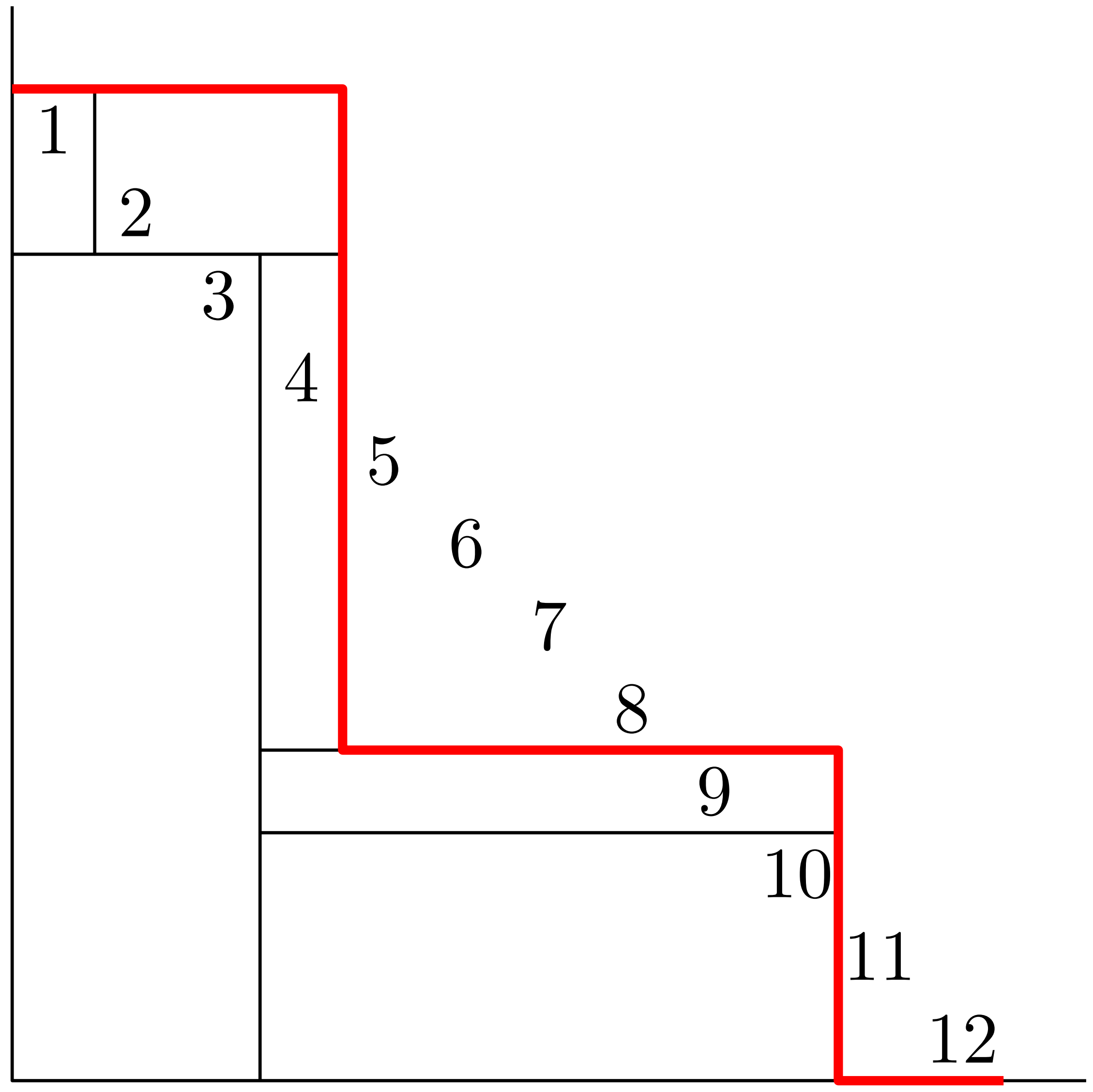

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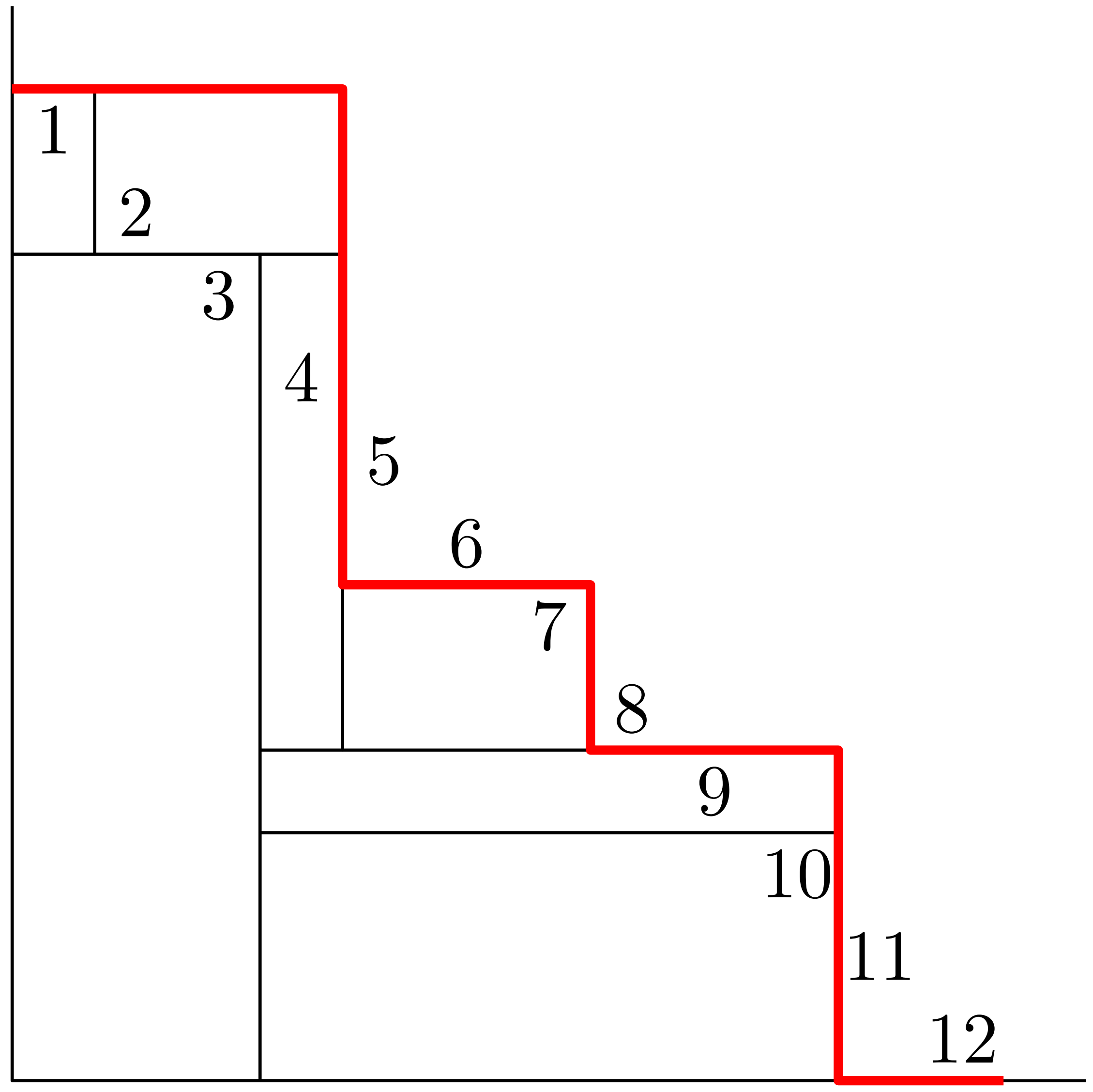

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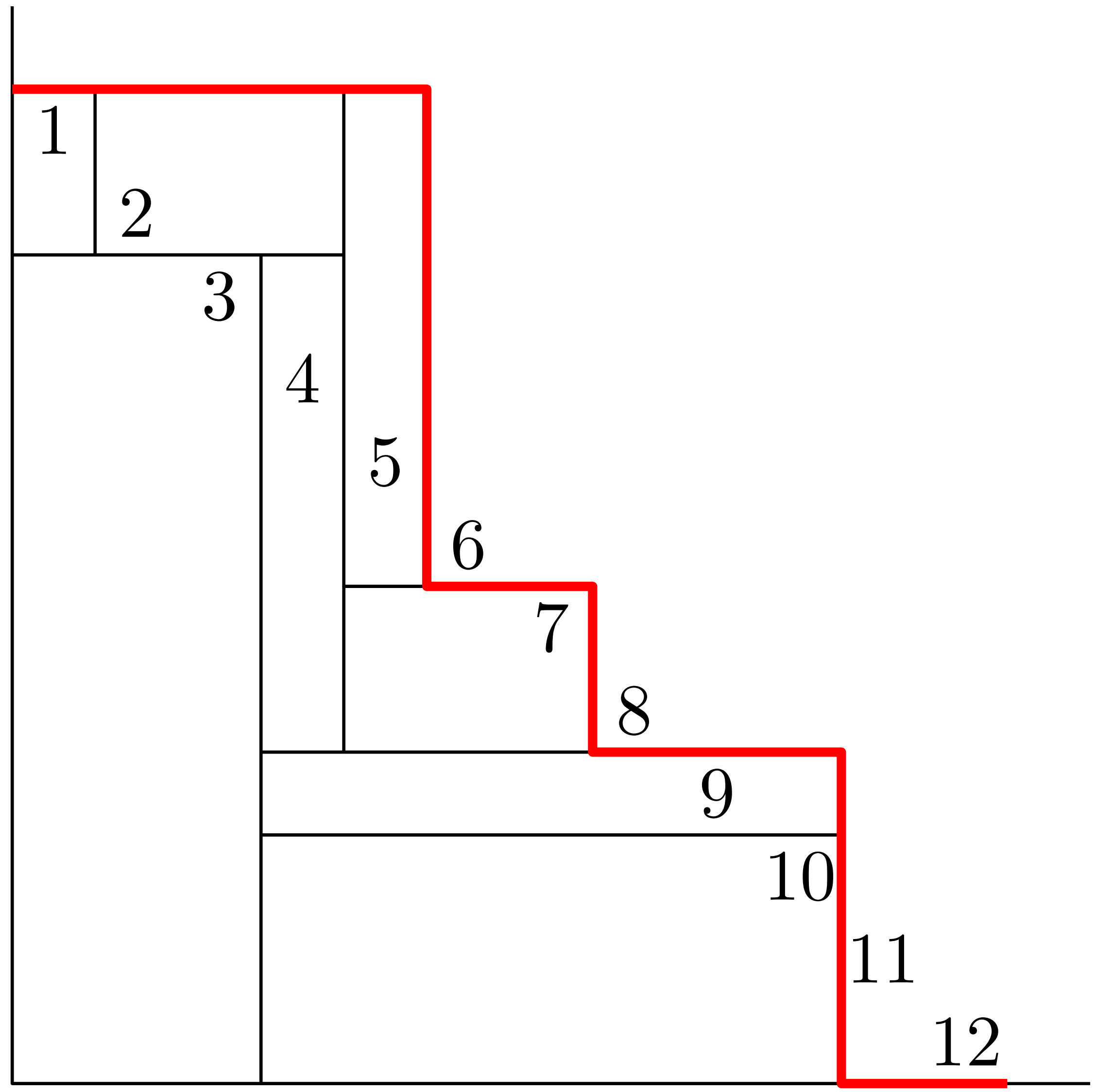

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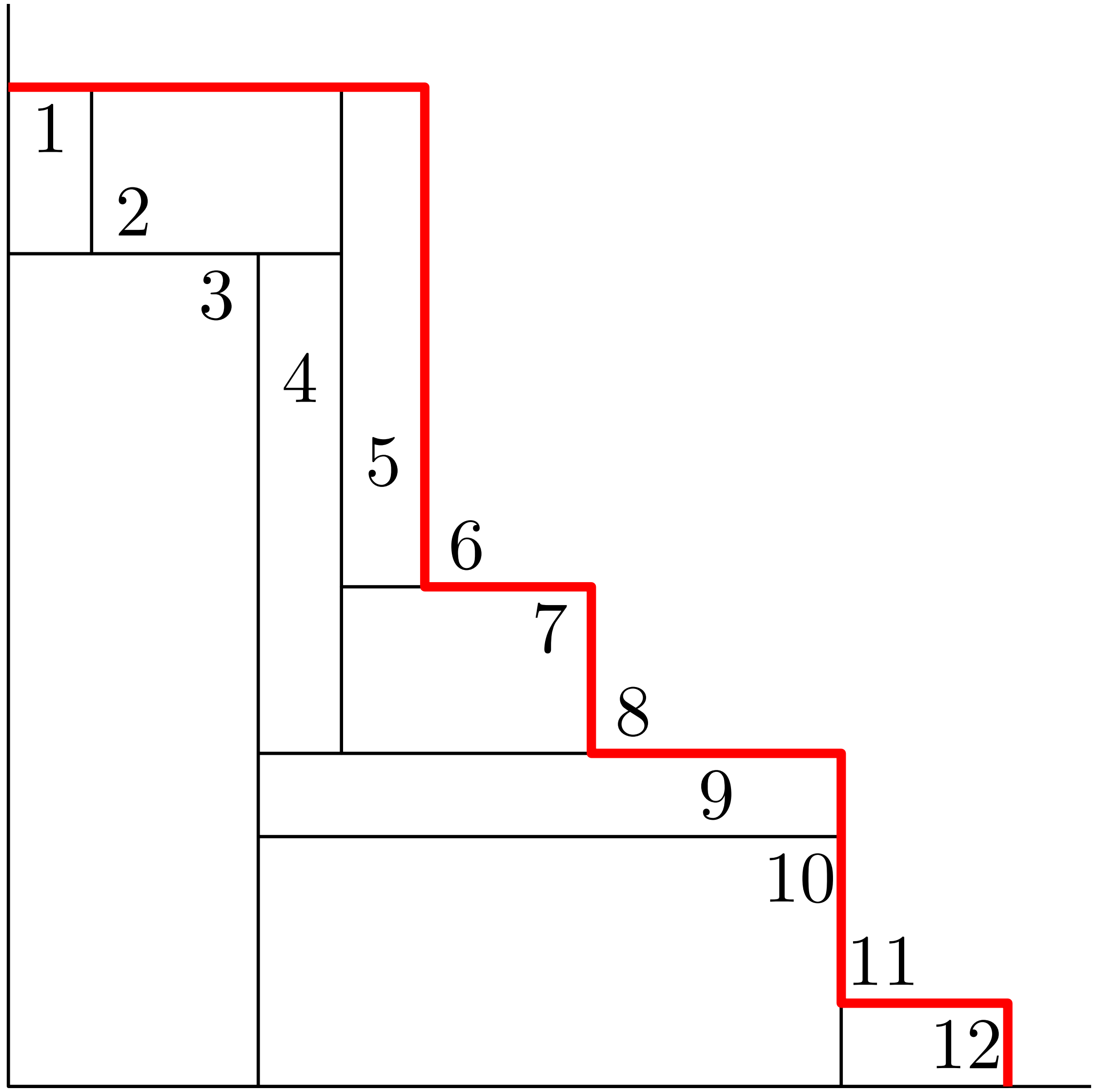

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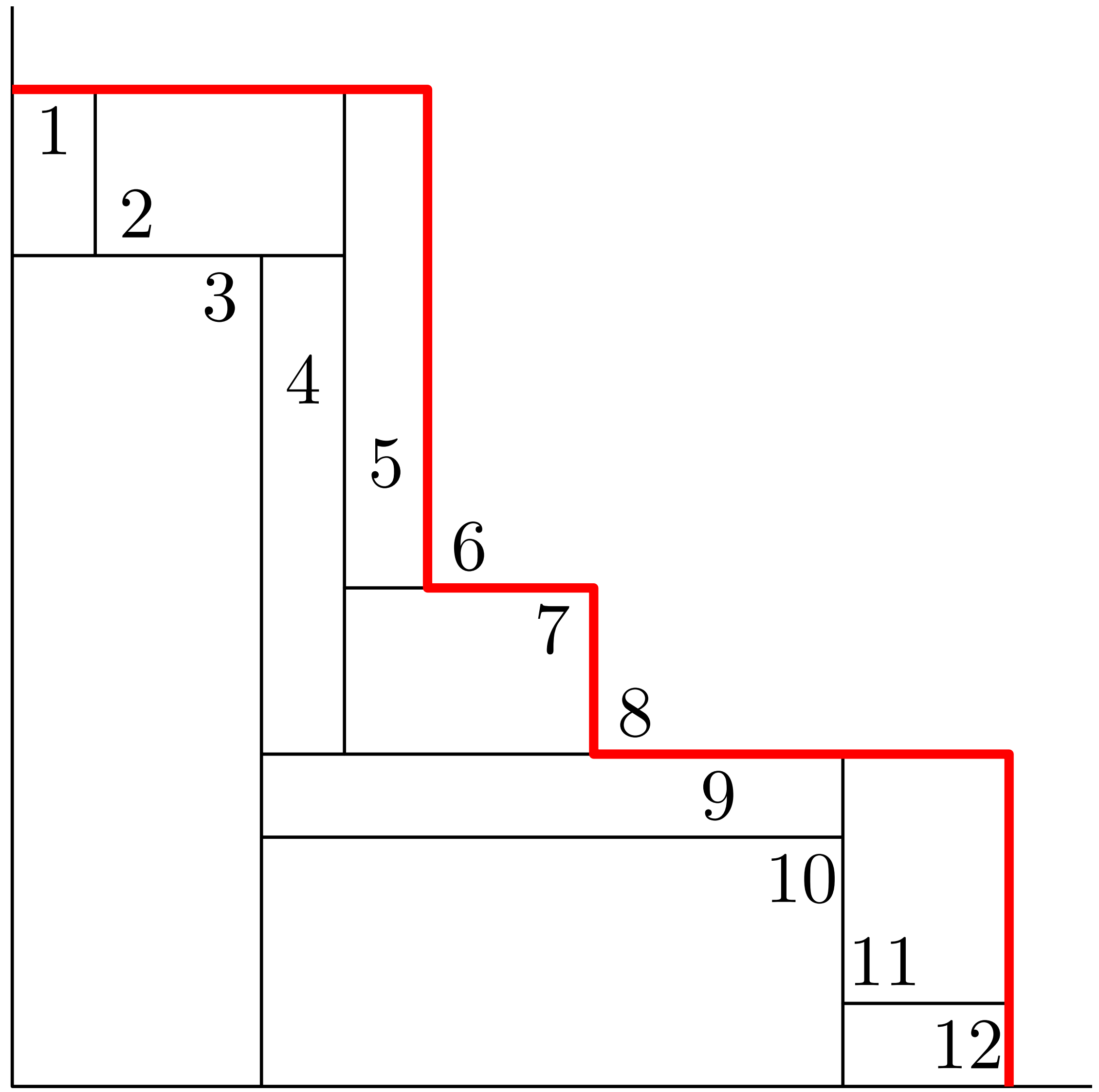

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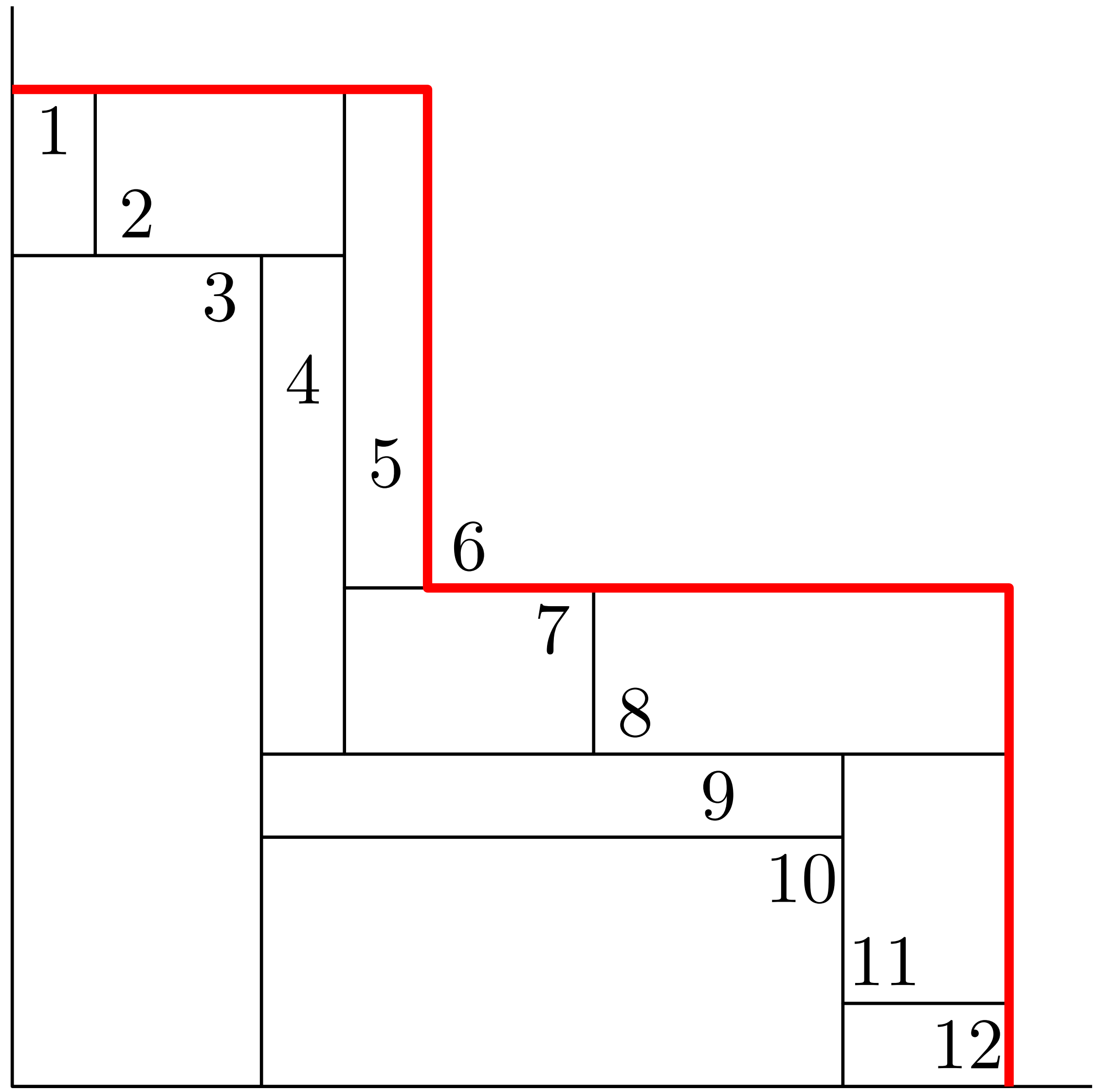

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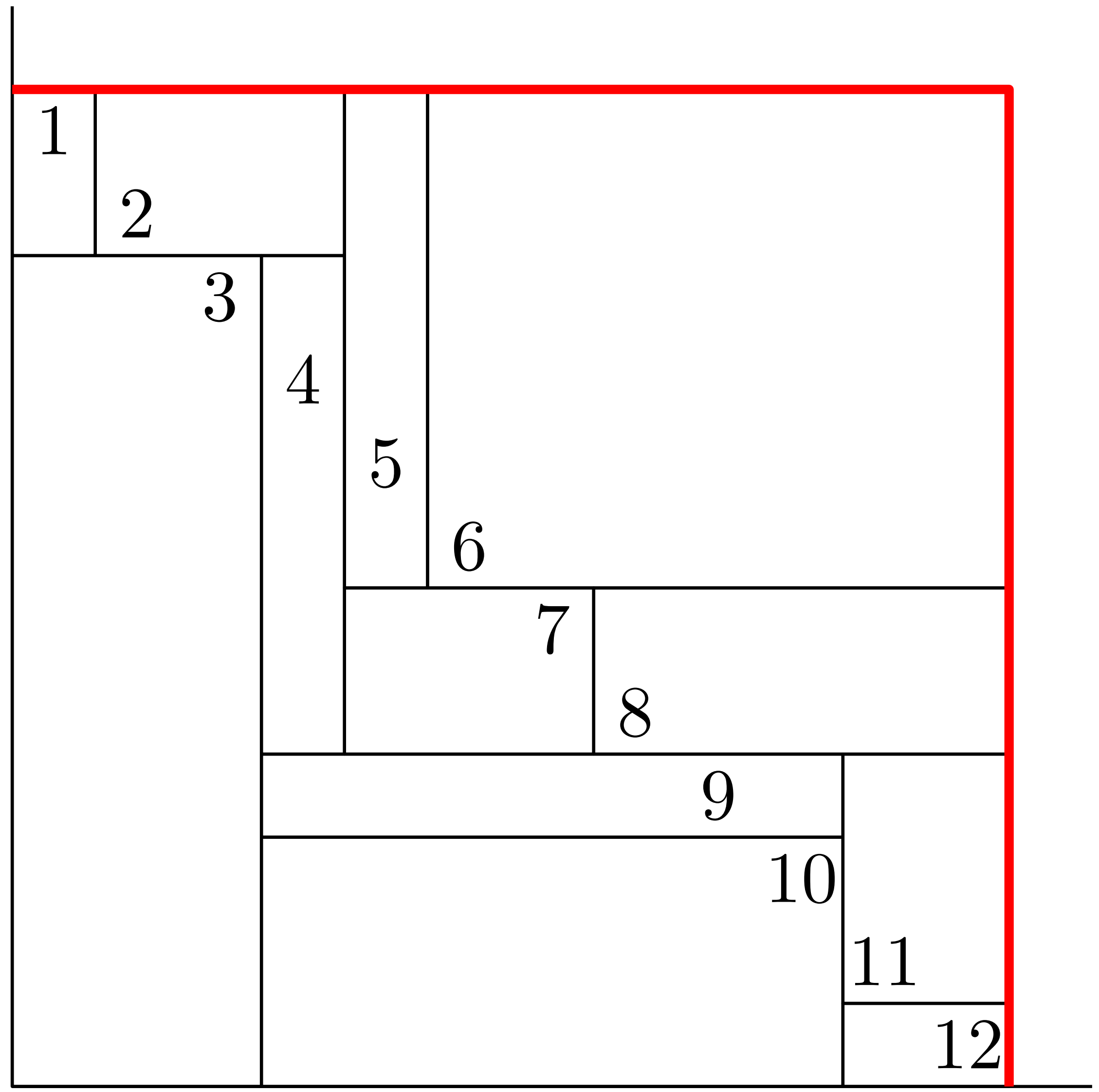

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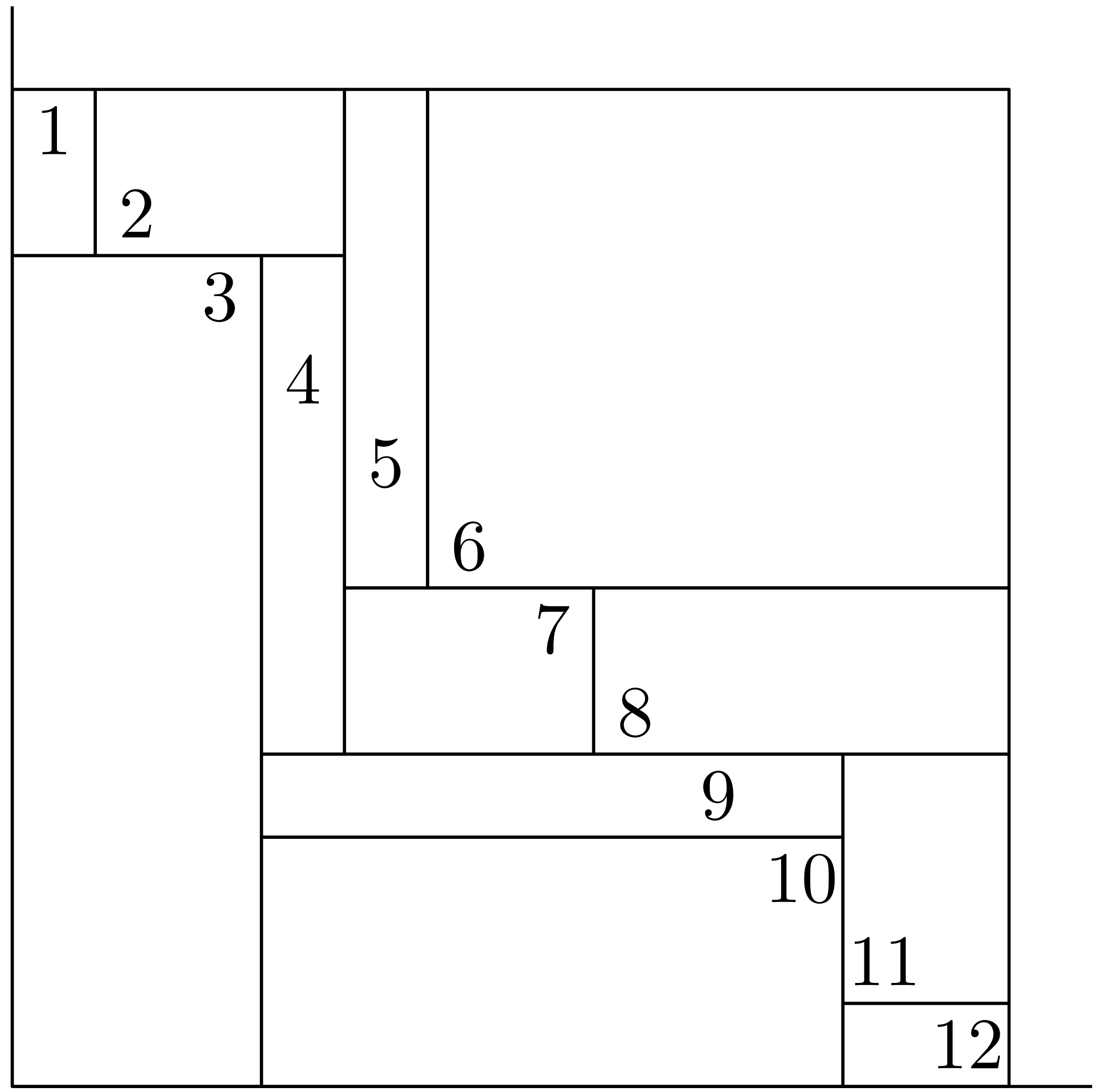

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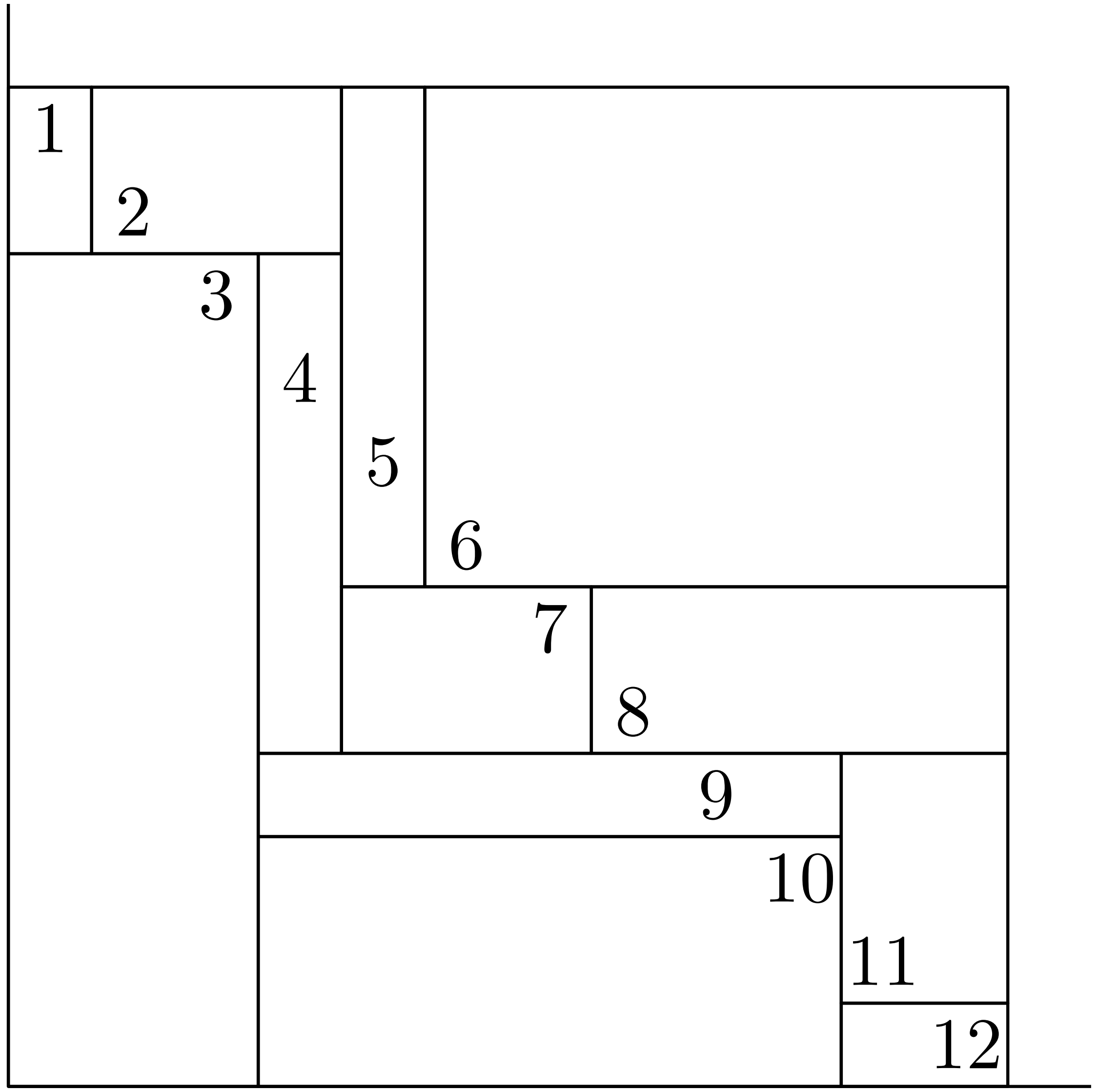

Theorem (Law and Reading 2012)

The map γ_w is a bijection when restricted to Baxter permutations (Av($2\underline{41}3$, $3\underline{14}2$)).

Characterizing Rectangulation Pattern Avoidance with Permutation Patterns

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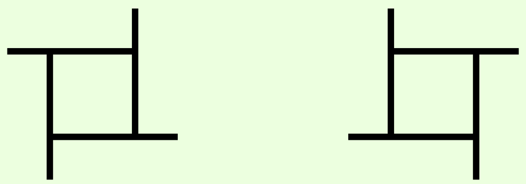
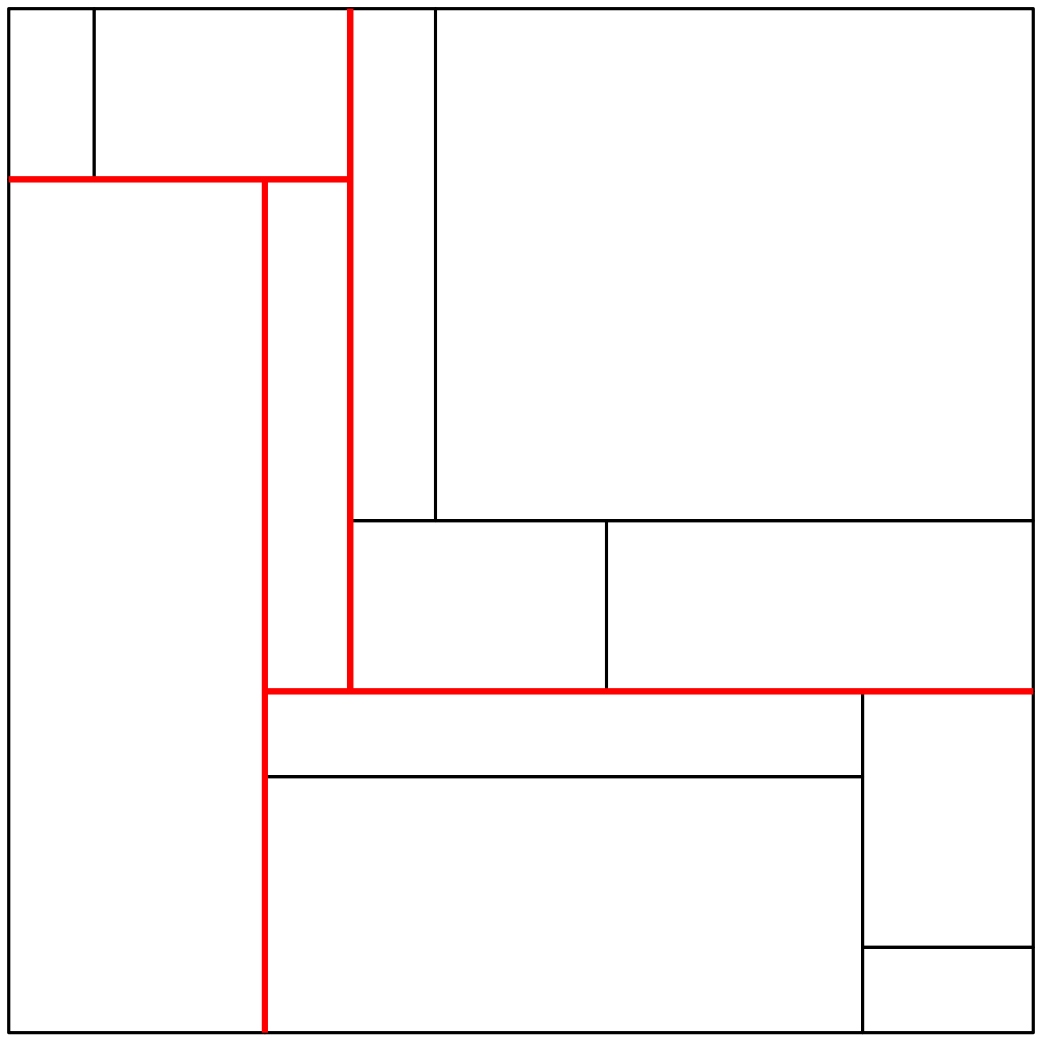
A rectangulation is *guillotine* if it avoids the following two patterns (called "windmills"):





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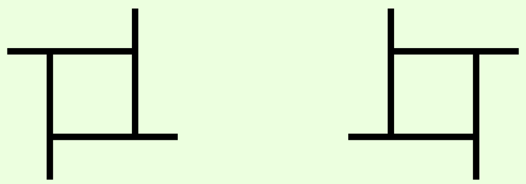
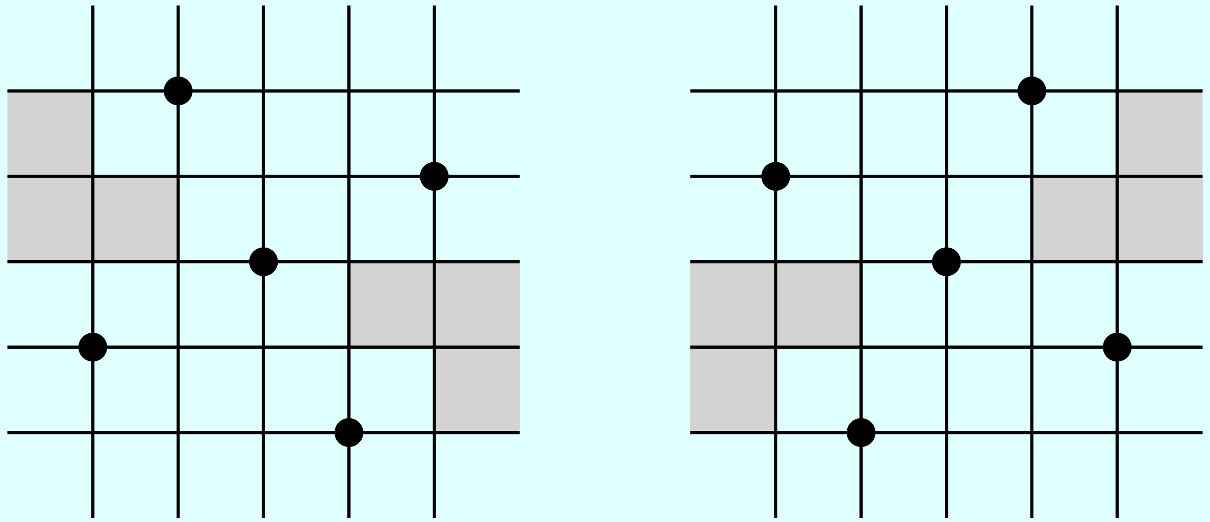


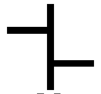
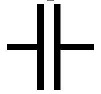
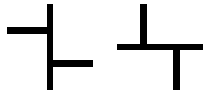
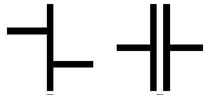
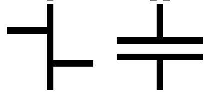
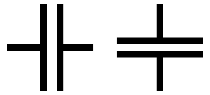
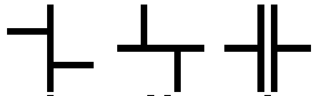
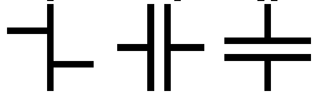
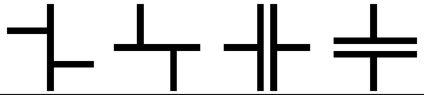
Theorem (Asinowski, Cardinal, Felsner, Fusy 2024)

Let $\pi \in S_n$. Then $\gamma_w(\pi)$ is guillotine if and only if π avoids both of the following mesh patterns:



Characterizing Rectangulation Pattern Avoidance with Permutation Patterns

Theorem (Asinowski and Banderier 2024)

Guillotine diagonal rectangulations avoiding...	Separable permutations avoiding...	G.f.	OEIS
$\varnothing$	$\varnothing$	alg.	A006318
	$2\underline{14}3$	alg.	A106228
	21354	alg.	A363809
	$2\underline{14}3, 3\underline{41}2$	alg.	A078482
	2143	alg.	A033321
	$2\underline{14}3, 45312$	alg.	A363810
	21354, 45312	rat.	A363811
	$2143, 3\underline{41}2$	alg.	A363812
	2143, 45312	rat.	A363813
	2143, 3412	rat.	A006012

Characterizing Rectangulation Pattern Avoidance with Permutation Patterns

Can we always characterize rectangulation pattern avoidance with permutation patterns?

Characterizing Rectangulation Pattern Avoidance with Permutation Patterns

Can we always characterize rectangulation pattern avoidance with permutation patterns? **No**

Characterizing Rectangulation Pattern Avoidance with Permutation Patterns

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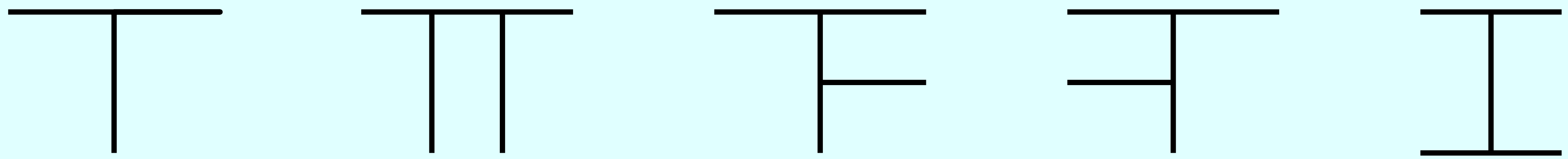
Theorem (Asinowski, Mütze, Namrata, P. 2025^{+})

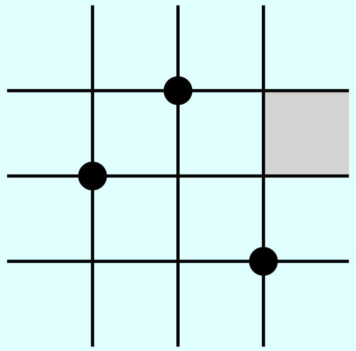
We can characterize all of the following rectagulation patterns as permutation mesh patterns (along with their reflections):



Mesh Pattern for $R^w(\top)$

Theorem (Asinowski, Mütze, Namrata, P. 2025^+)

Let $\pi \in S_n$. Then $\gamma_w(\pi)$ avoids $\top$ if and only if π avoids the following mesh pattern:



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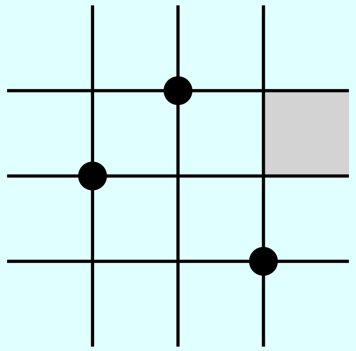

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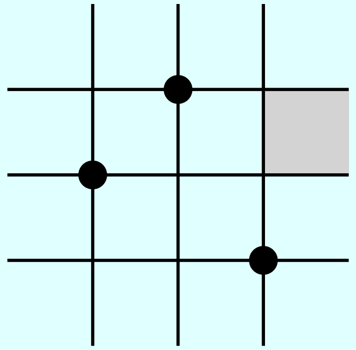
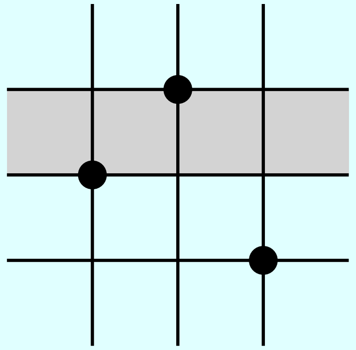
τ :  τ' : 

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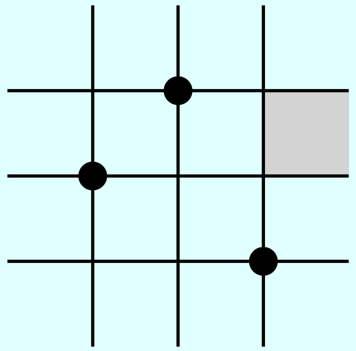
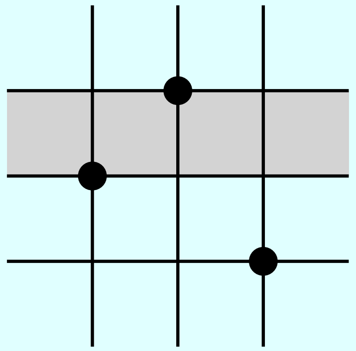
τ : 

τ' : 

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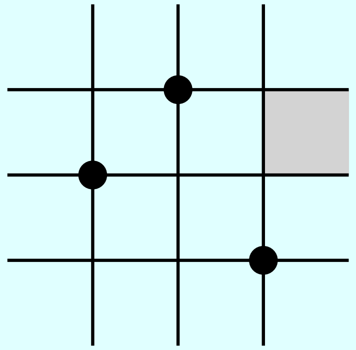
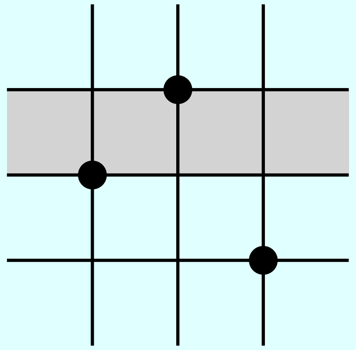
τ :  τ' : 

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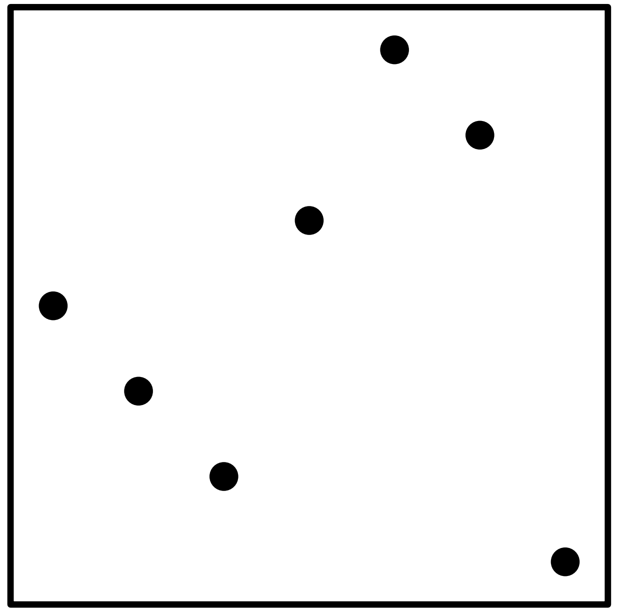

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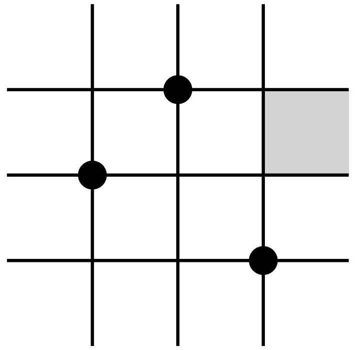

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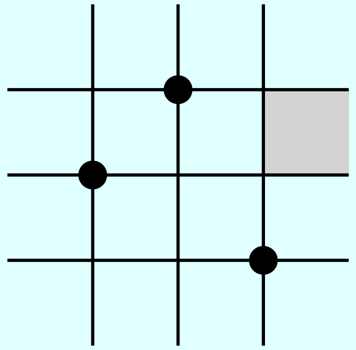
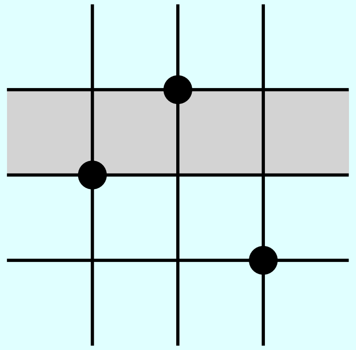
τ : 

τ' : 

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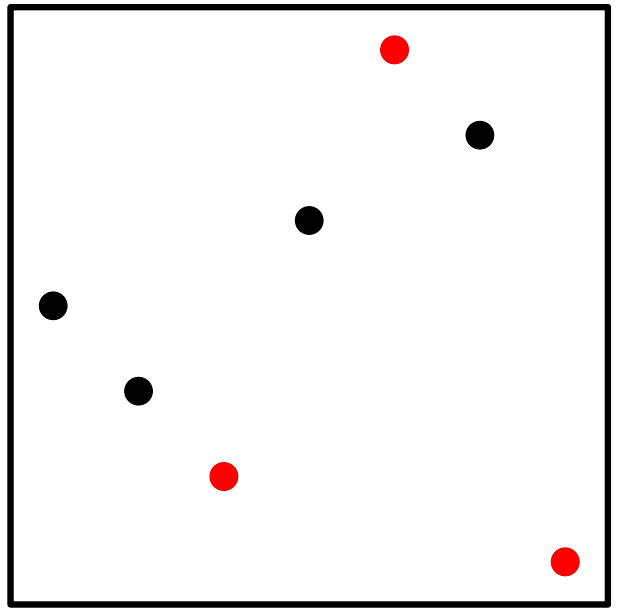

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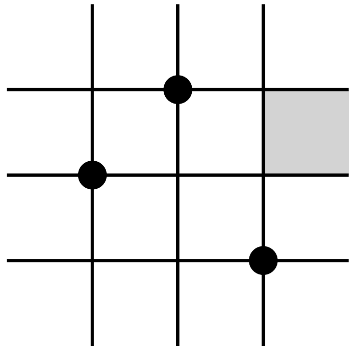

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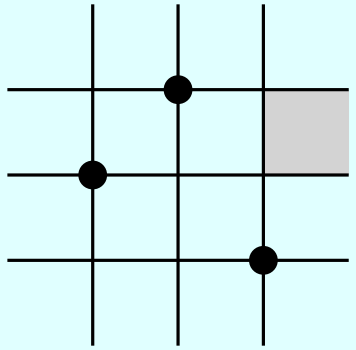
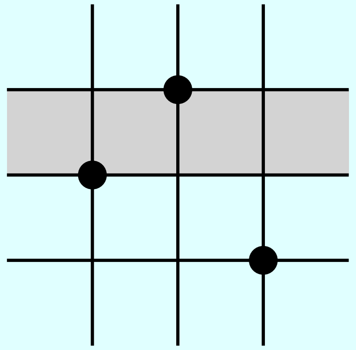
τ : 

τ' : 

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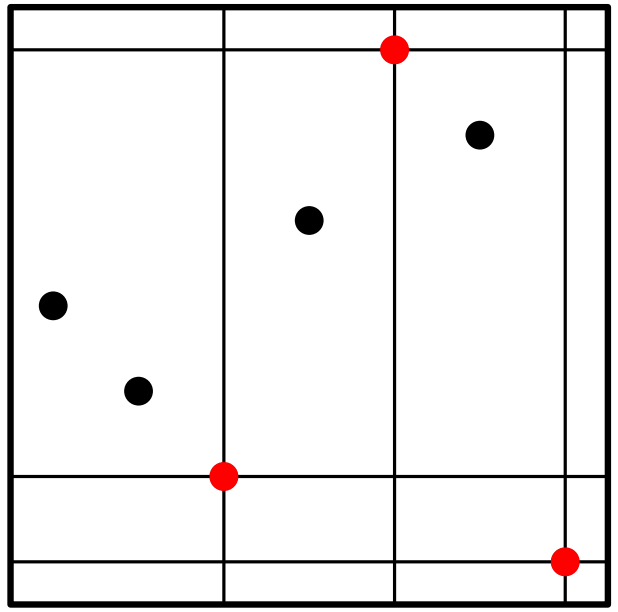

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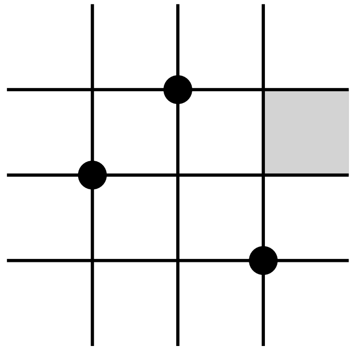

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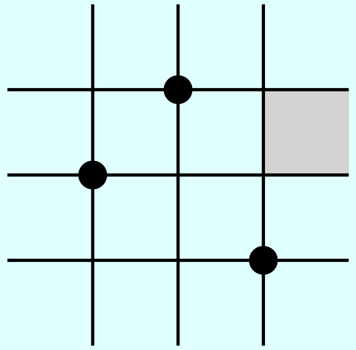
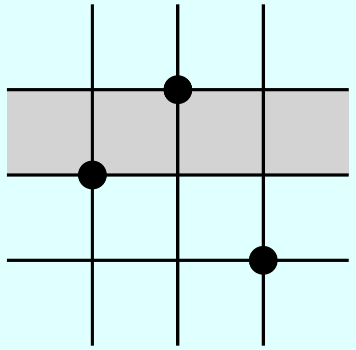
τ :  τ' : 

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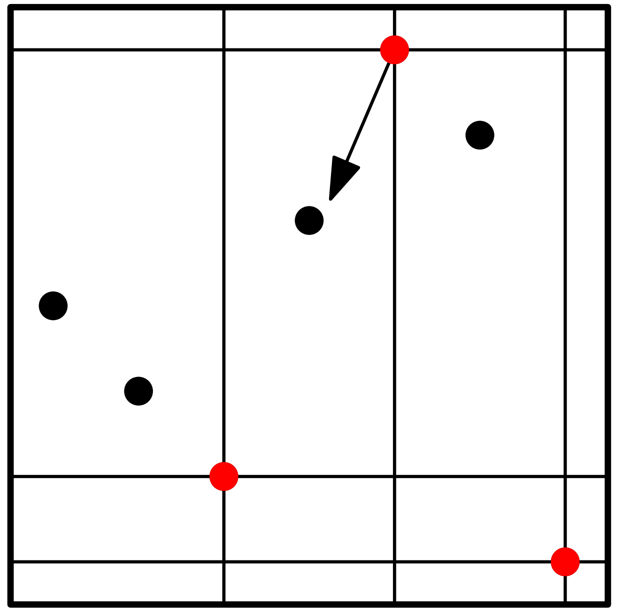

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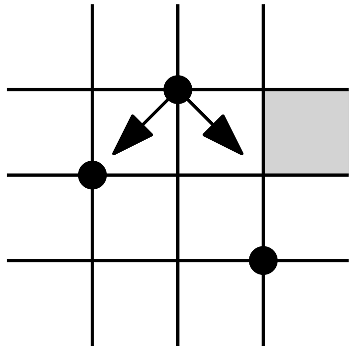

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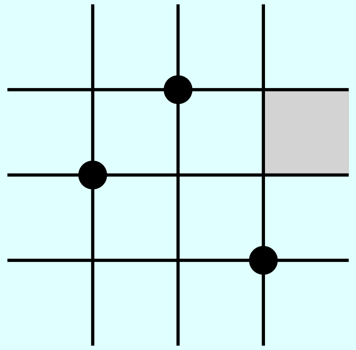
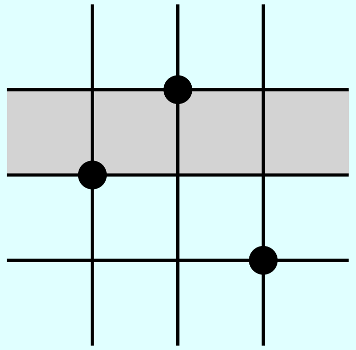
τ : 

τ' : 

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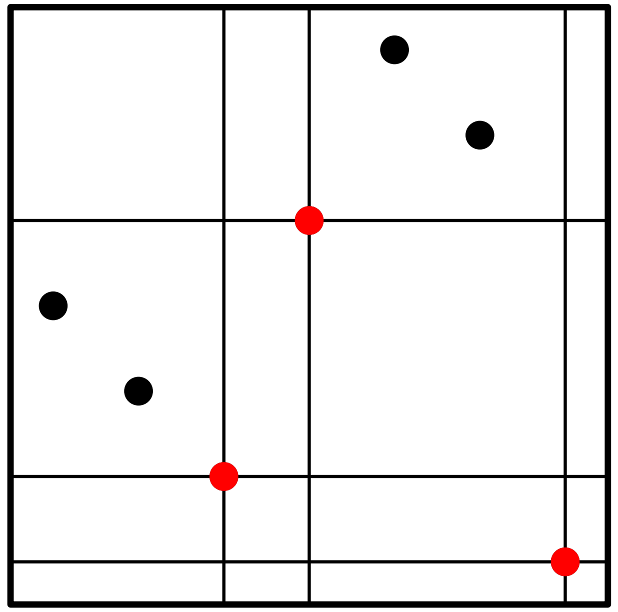

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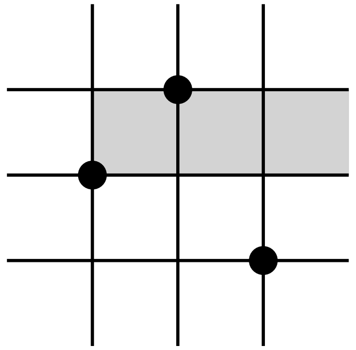

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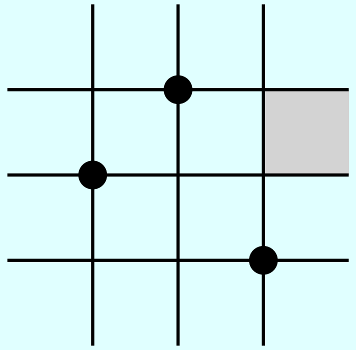
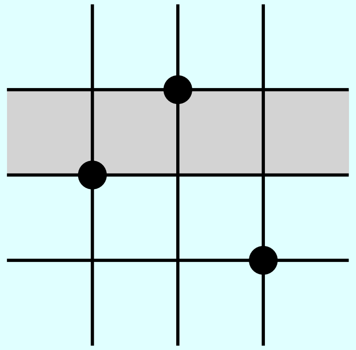
τ :  τ' : 

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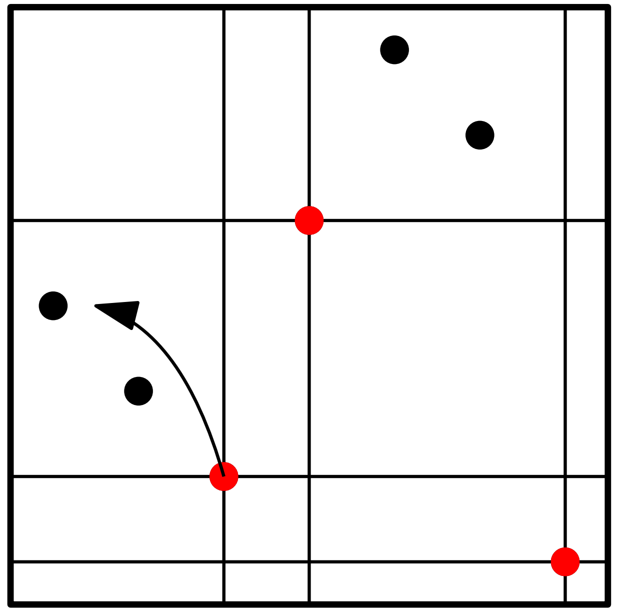

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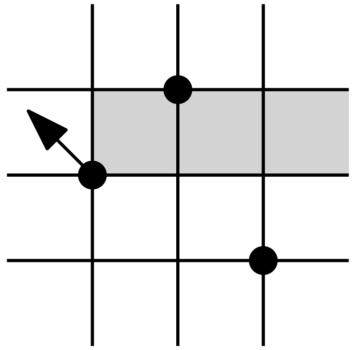

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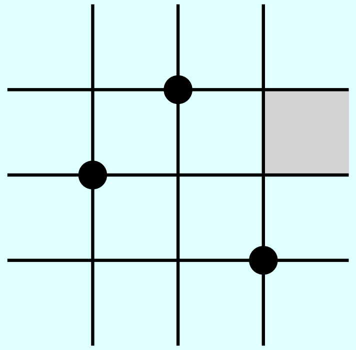
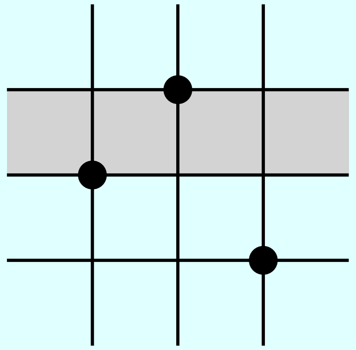
τ : 

τ' : 

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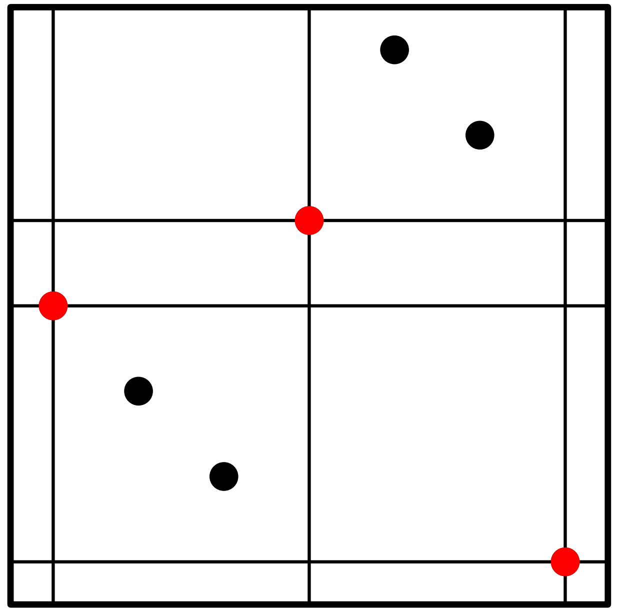

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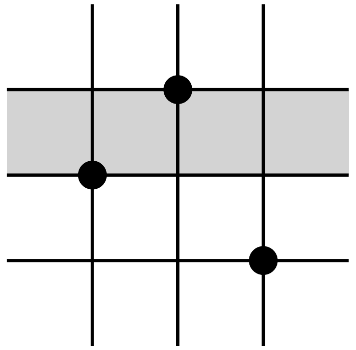

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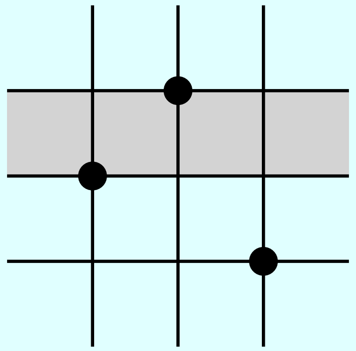

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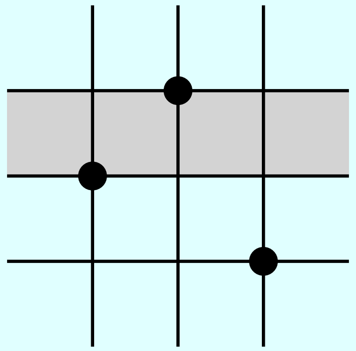

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$$a < b < c$$
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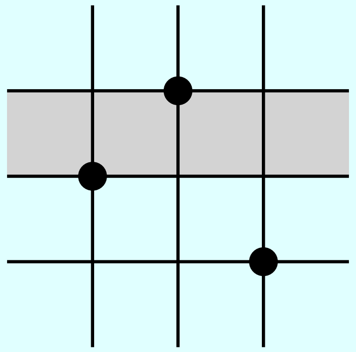

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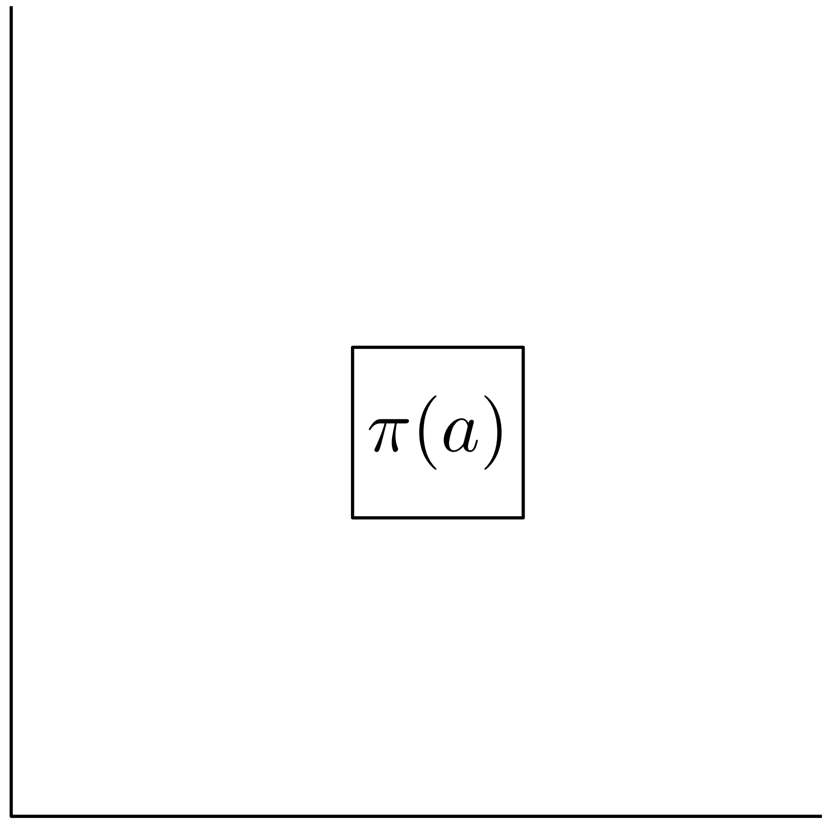

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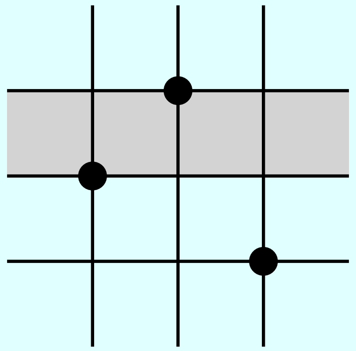

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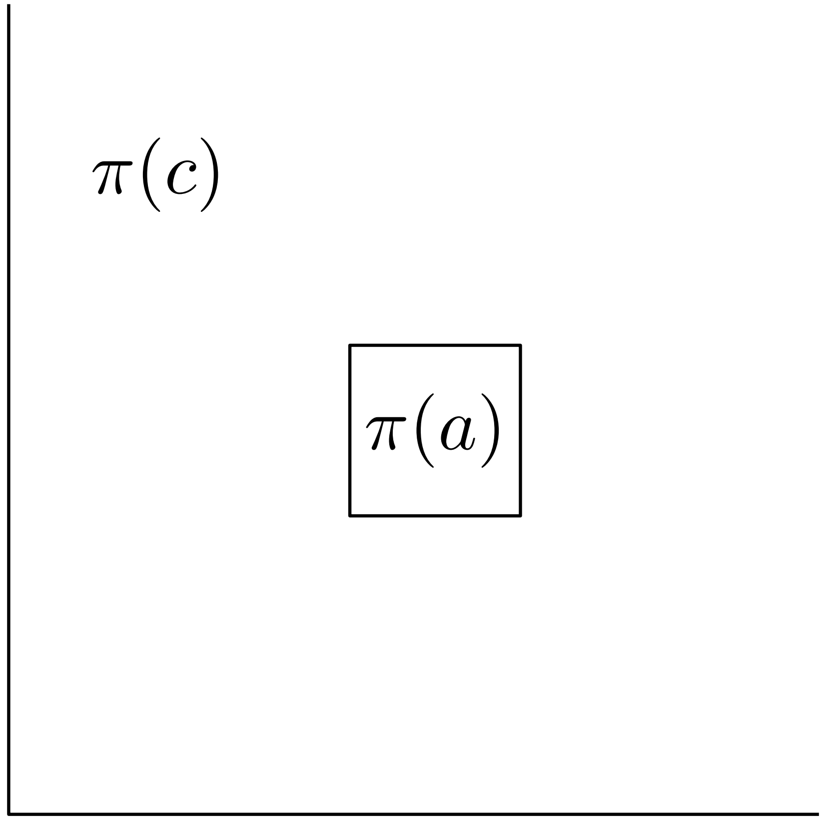

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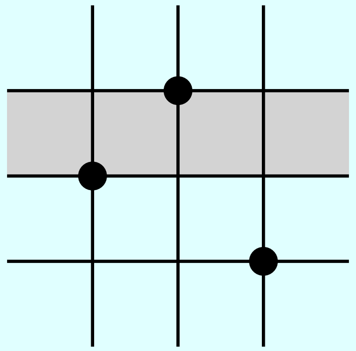

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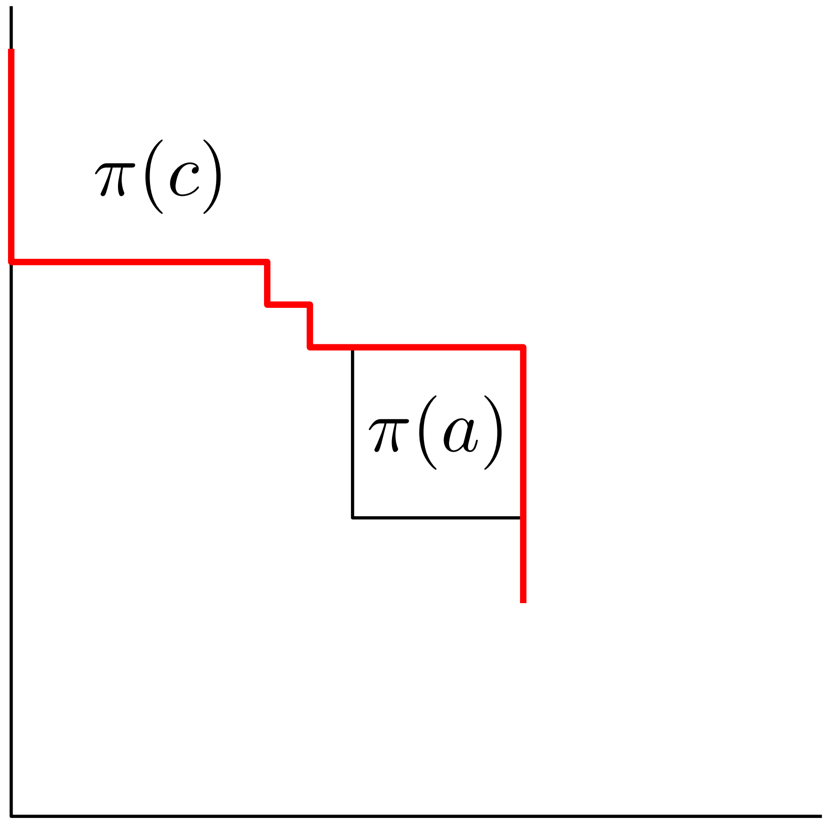

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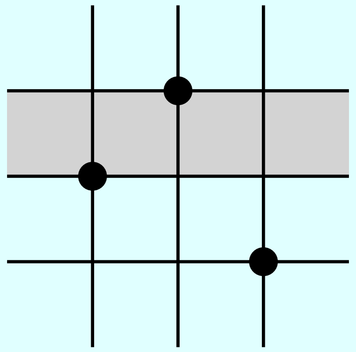

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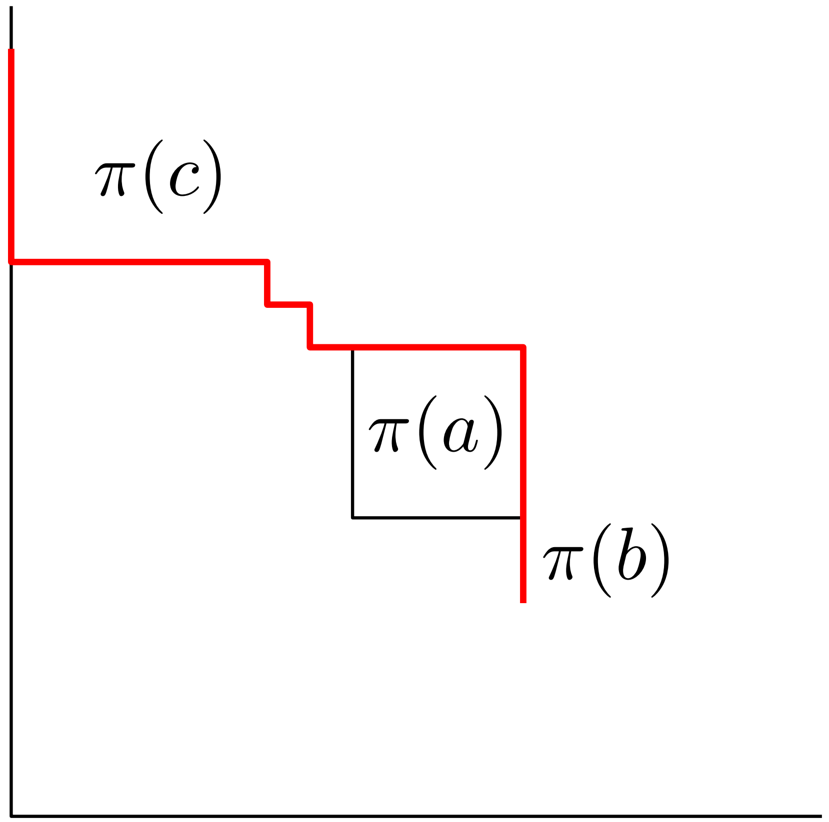

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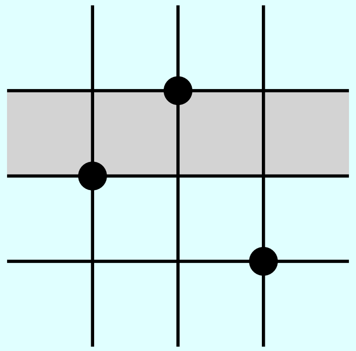

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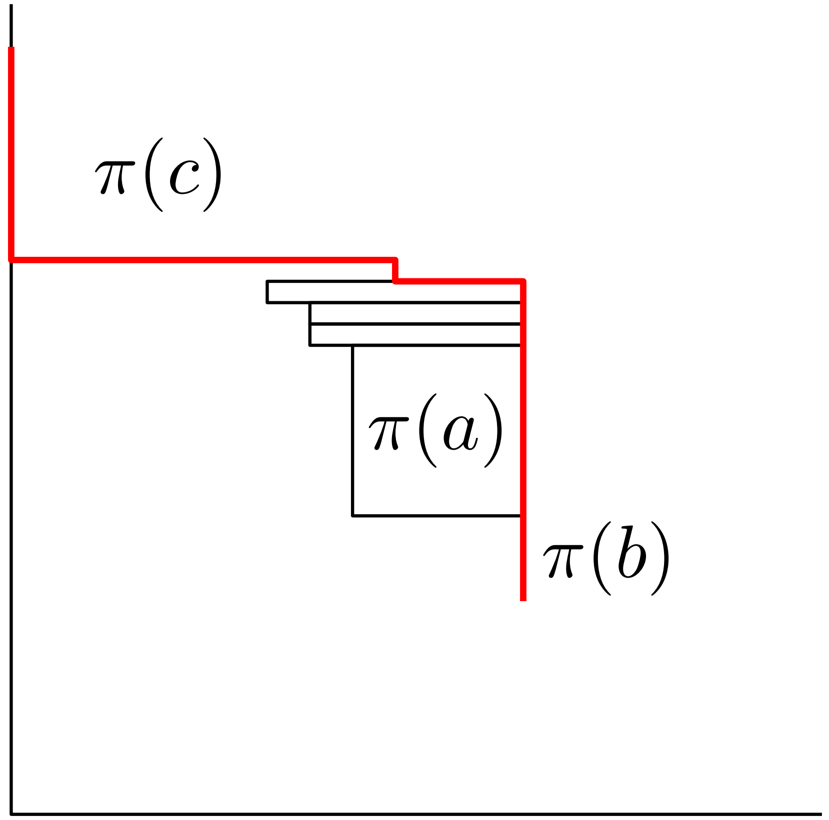

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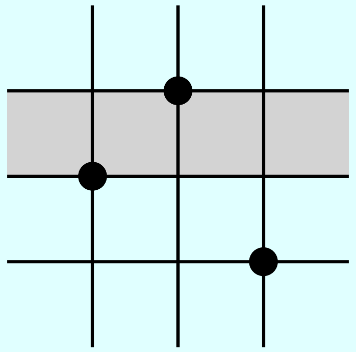

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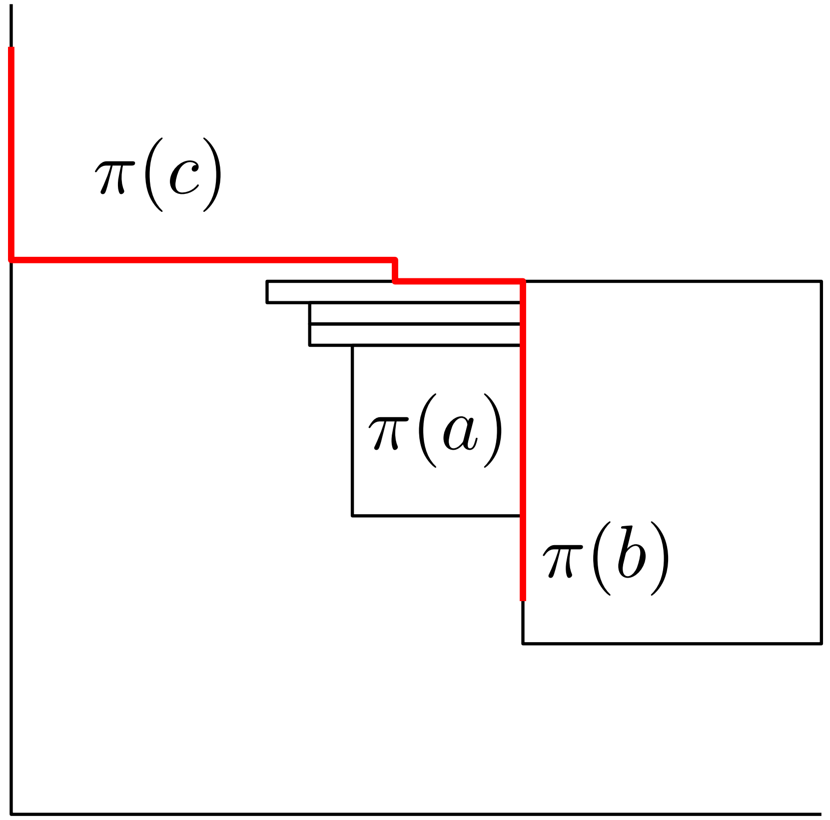
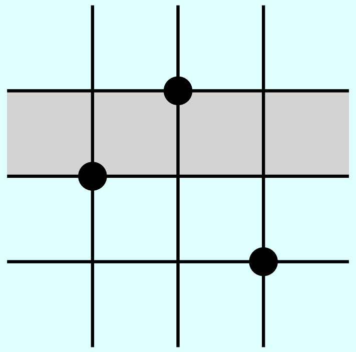


$$a < b < c$$
$$\pi(c) < \pi(a) < \pi(b) = \pi(a) + 1$$

Mesh Pattern for $R^w(\top)$

Theorem (Asinowski, Mütze, Namrata, P. 2025$^+$)

Let $\pi \in S_n$. Then $\gamma_w(\pi)$ avoids $\top$ if and only if π avoids the following mesh pattern:



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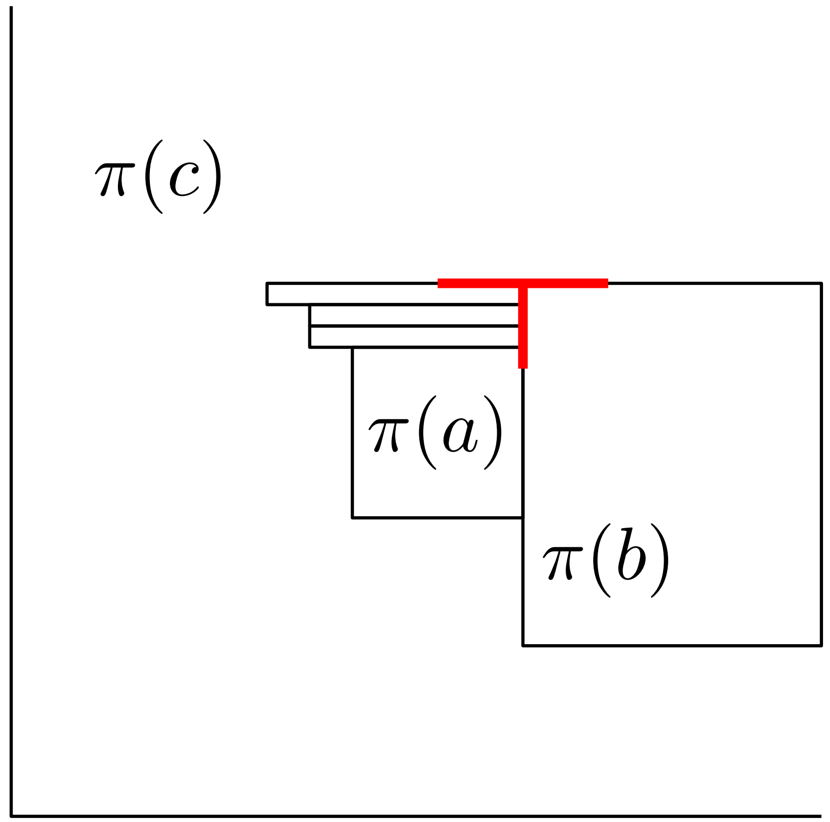

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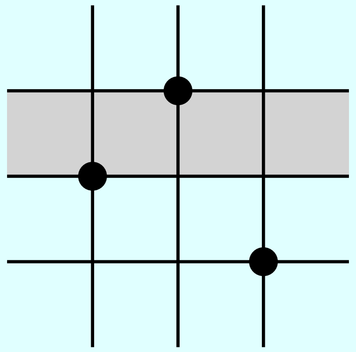
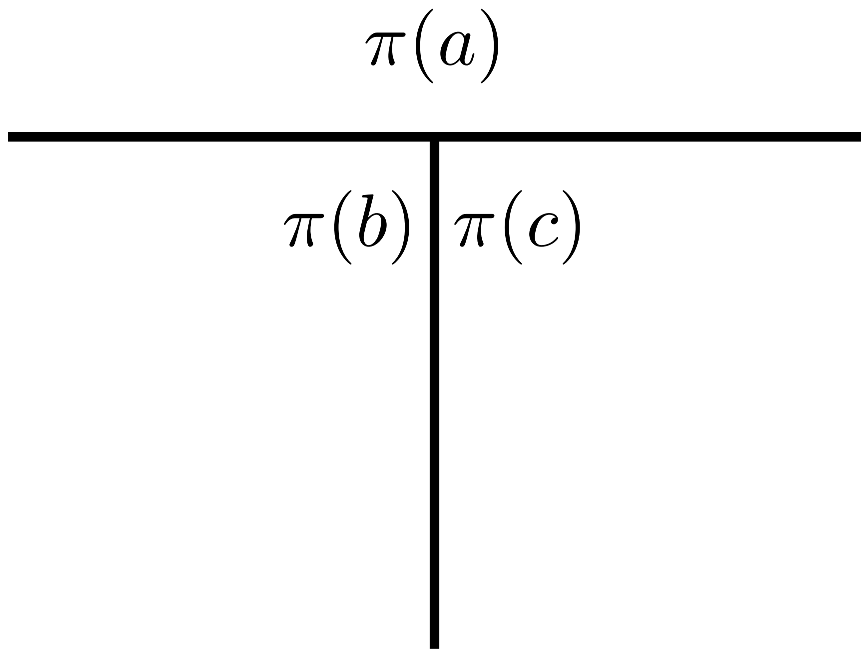


Proof:

$\Leftarrow$ We will show that if $\gamma_w(\pi)$ contains $\top$, then π contains the mesh pattern.



Mesh Pattern for $R^w(\top)$

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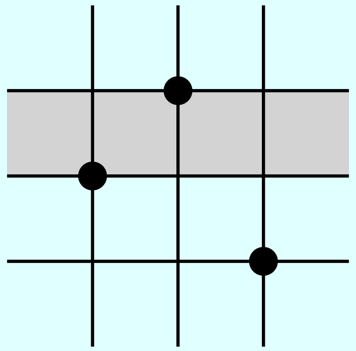
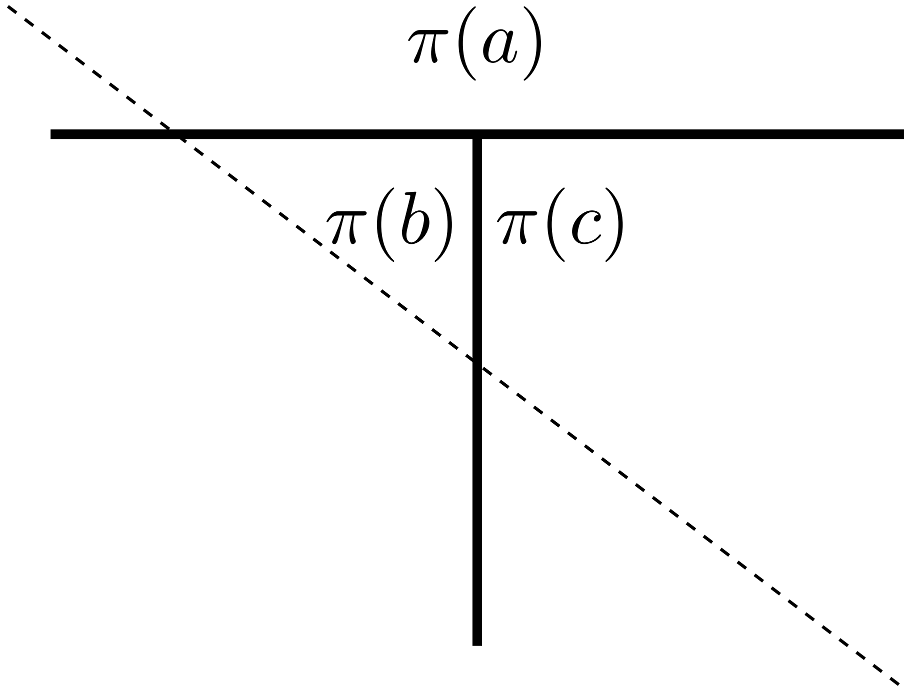


Proof:

$\Leftarrow$ We will show that if $\gamma_w(\pi)$ contains $\top$, then π contains the mesh pattern.

$\pi(a) < \pi(b) < \pi(c)$



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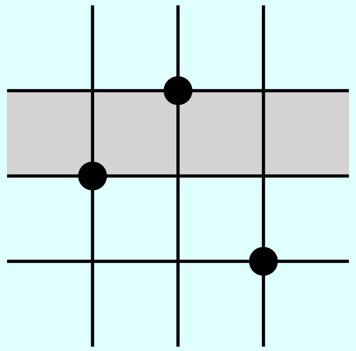

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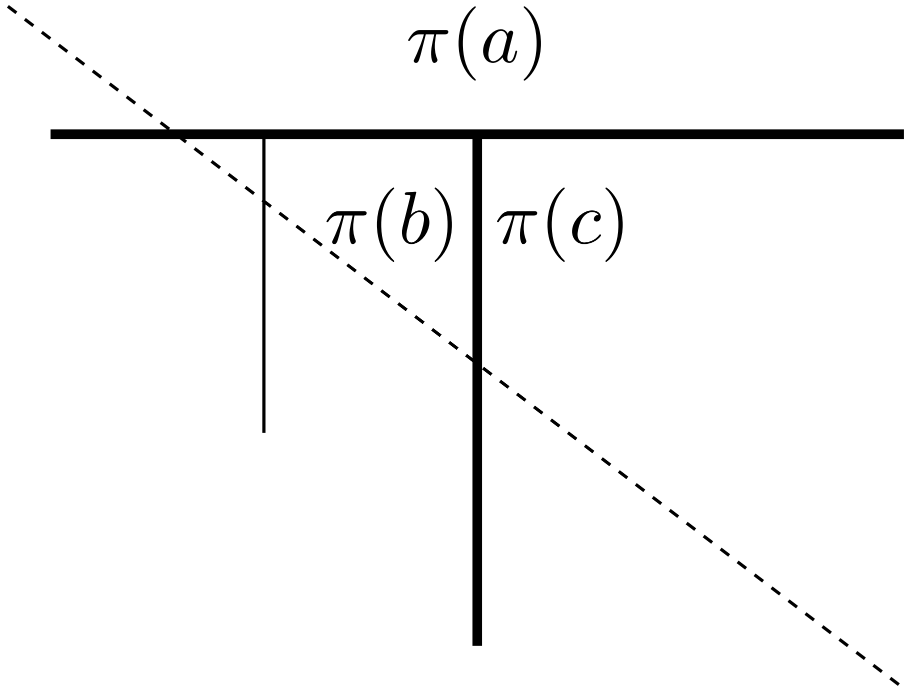
$\pi(a) < \pi(b) < \pi(c)$



Mesh Pattern for $R^w(\top)$

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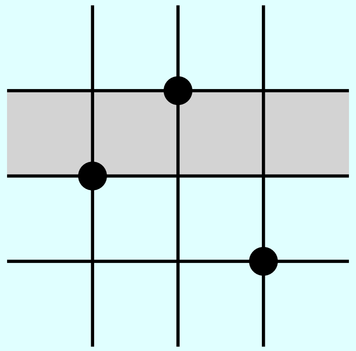
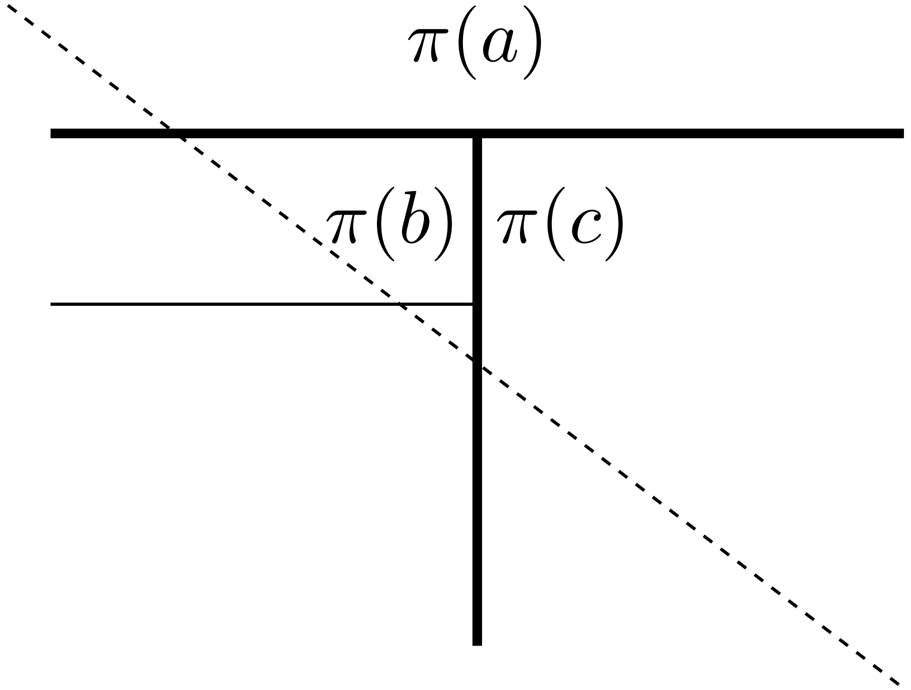


Proof:

$\Leftarrow$ We will show that if $\gamma_w(\pi)$ contains $\top$, then π contains the mesh pattern.

$\pi(a) < \pi(b) < \pi(c)$



Mesh Pattern for $R^w(\top)$

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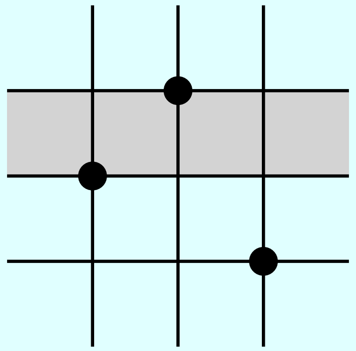
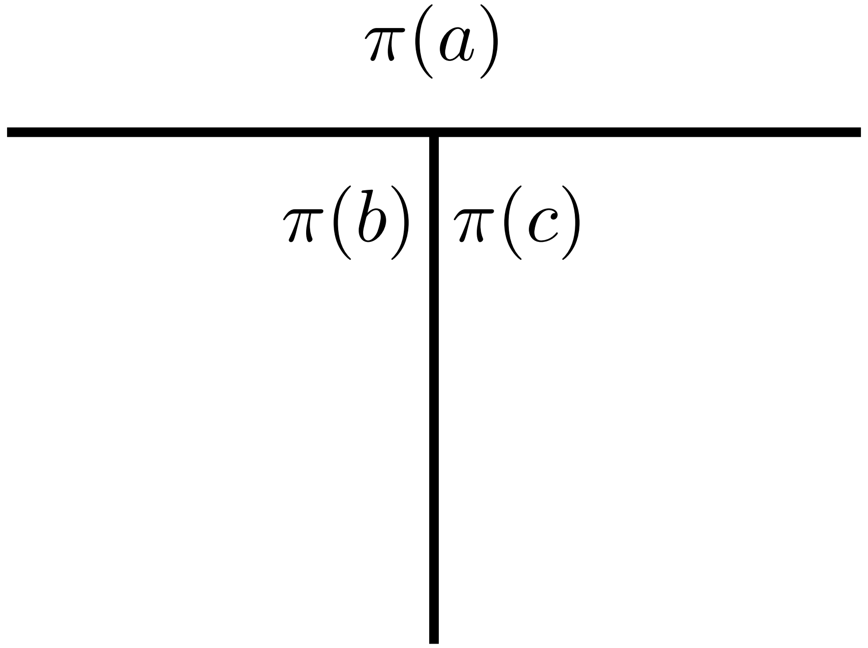


Proof:

$\Leftarrow$ We will show that if $\gamma_w(\pi)$ contains $\top$, then π contains the mesh pattern.

$$\pi(a) < \pi(b) < \pi(c)$$
$$b < c < a$$



Mesh Pattern for $R^w(\top)$

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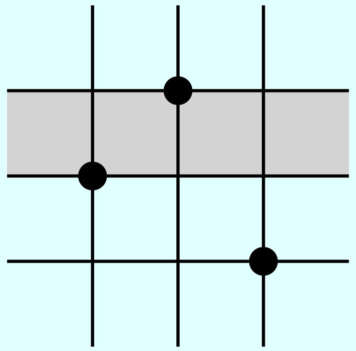
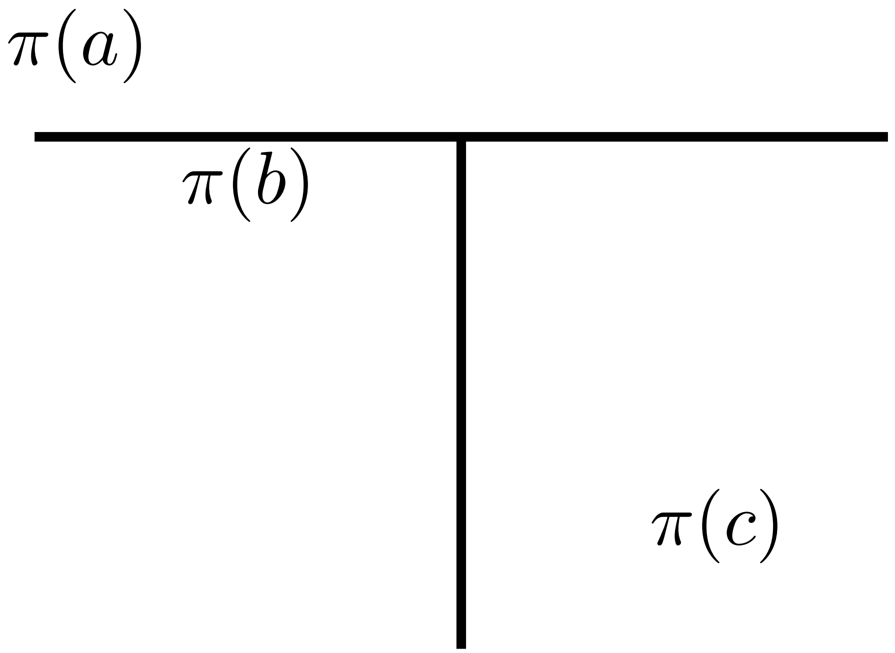


Proof:

$\Leftarrow$ We will show that if $\gamma_w(\pi)$ contains $\top$, then π contains the mesh pattern.

$$\pi(a) < \pi(b) < \pi(c)$$

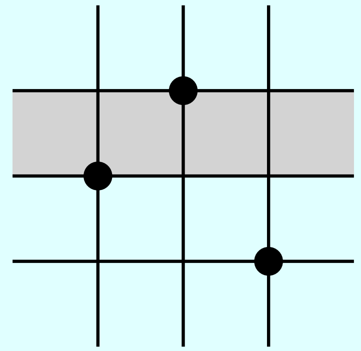
$$b < c < a$$



Mesh Pattern for $R^w(\top)$

Theorem (Asinowski, Mütze, Namrata, P. 2025$^+$)

Let $\pi \in S_n$. Then $\gamma_w(\pi)$ avoids $\top$ if and only if π avoids the following mesh pattern:

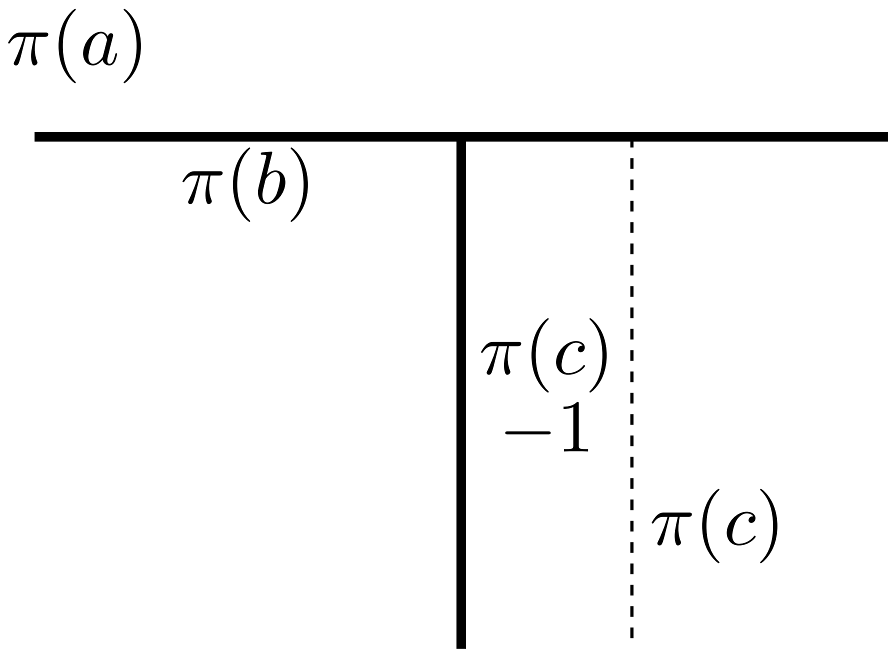


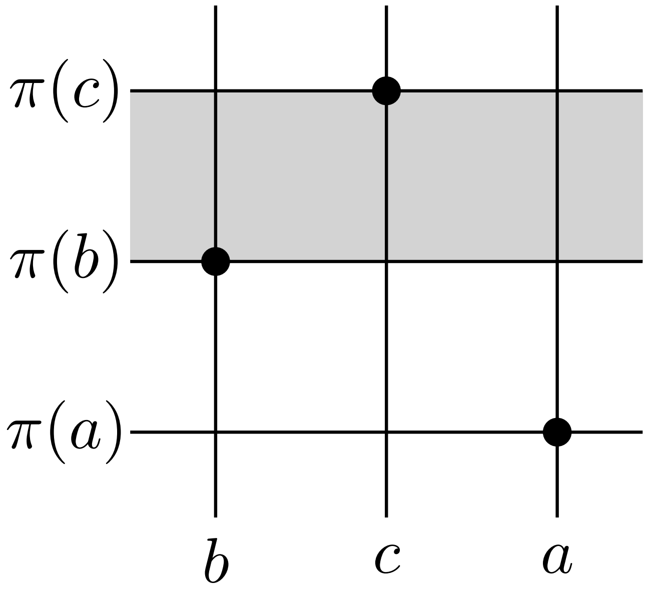
Proof:

$\Leftarrow$ We will show that if $\gamma_w(\pi)$ contains $\top$, then π contains the mesh pattern.

$$\pi(c) - 1 = \pi(b)$$

$$\pi(a) < \pi(b) < \pi(c)$$
$$b < c < a$$

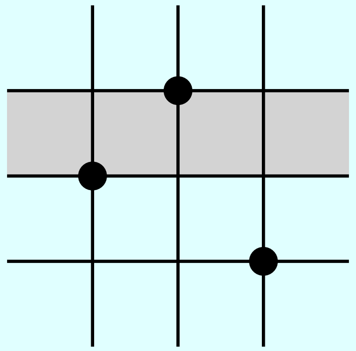




Mesh Pattern for $R^w(\top)$

Theorem (Asinowski, Mütze, Namrata, P. 2025$^+$)

Let $\pi \in S_n$. Then $\gamma_w(\pi)$ avoids $\top$ if and only if π avoids the following mesh pattern:



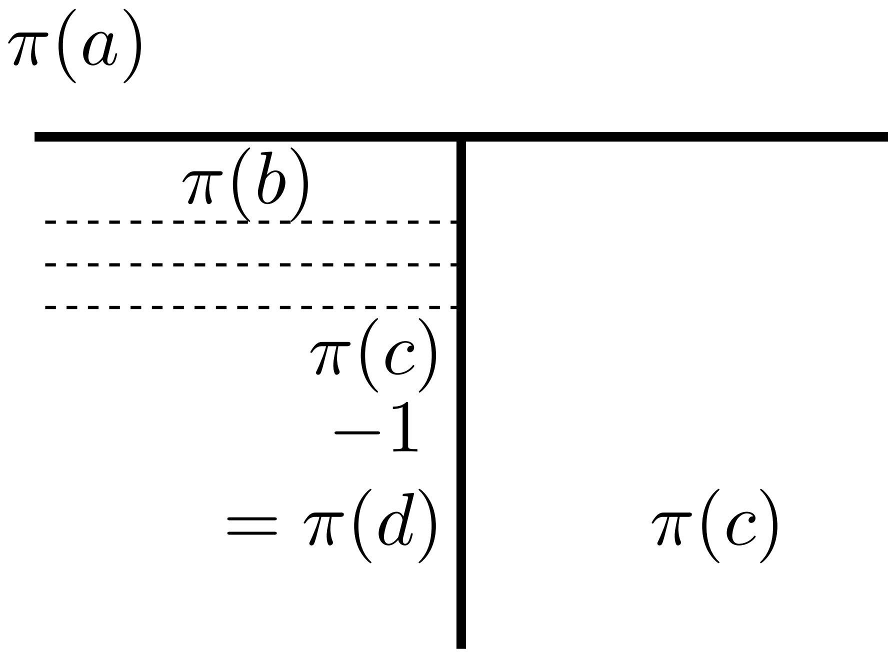
Proof:

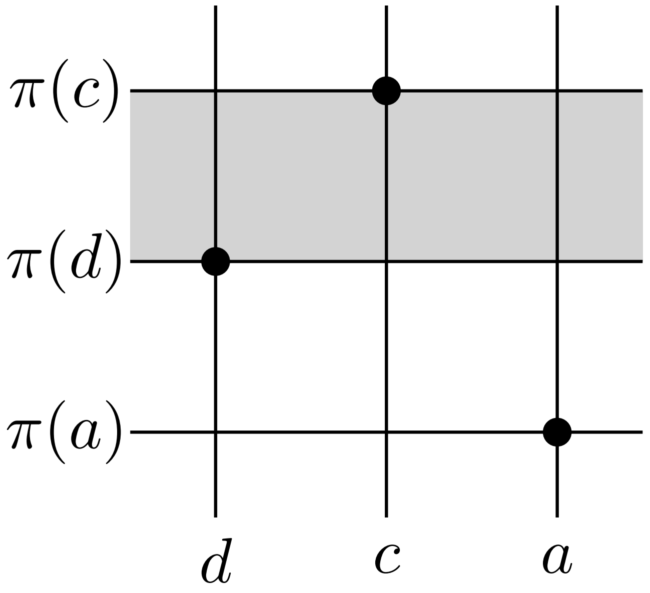
$\Leftarrow$ We will show that if $\gamma_w(\pi)$ contains $\top$, then π contains the mesh pattern.

$$\pi(c) - 1 = \pi(d)$$

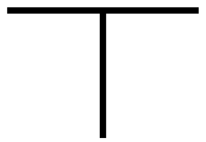
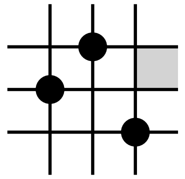
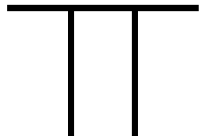
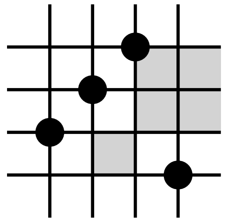
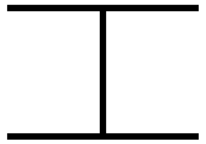
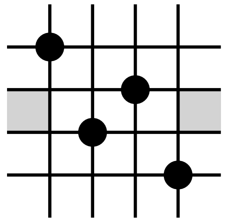
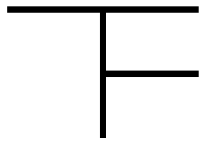
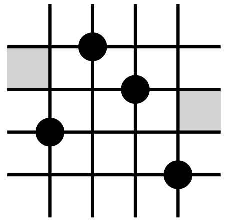
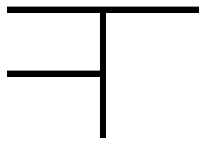
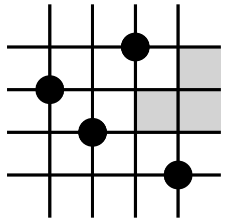
$$\pi(a) < \pi(b) < \pi(d) < \pi(c)$$

$$d < b < c < a$$





Summary of Results

Rectangulation Pattern	Mesh Pattern	w/s	Enumeration for $n = 1, \ldots, 10$	OEIS
		w	$1, 2, 5, 14, 42, 132, 429, 1430, 4862, 16796$	A000108
		s	$1, 2, 5, 15, 51, 189, 746, 3091, 13311, 59146$	A279555
		w	$1, 2, 6, 21, 81, 334, 1445, 6485, 29954, 141609$	
		s	$1, 2, 6, 23, 103, 514, 2785, 16097, 98030, 623323$	
		w	$1, 2, 6, 21, 79, 309, 1237, 5026, 20626, 85242$	A026737 & A111279*
		s	$1, 2, 6, 23, 101, 482, 2433, 12787, 69270, 384134$	
		w	$1, 2, 6, 21, 80, 322, 1347, 5798, 25512, 114236$	A106228*
		s	$1, 2, 6, 23, 102, 499, 2622, 14547, 84229, 504775$	
		w	$1, 2, 6, 21, 80, 322, 1347, 5798, 25512, 114236$	A106228*
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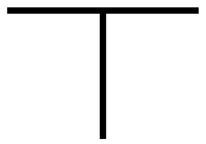
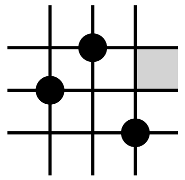
*Conjectured

Summary of Results

Rectangulation Pattern	Mesh Pattern	w/s	Enumeration for $n = 1, \ldots, 10$	OEIS
		w	$1, 2, 5, 14, 42, 132, 429, 1430, 4862, 16796$	A000108
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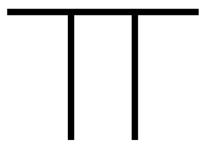	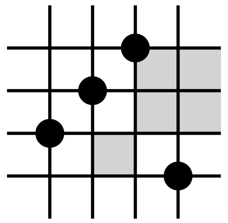	w	$1, 2, 6, 21, 81, 334, 1445, 6485, 29954, 141609$	
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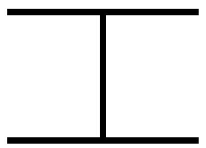	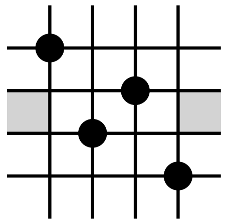	w	$1, 2, 6, 21, 79, 309, 1237, 5026, 20626, 85242$	A026737 & A111279*
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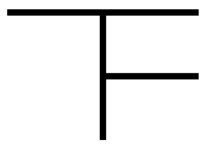
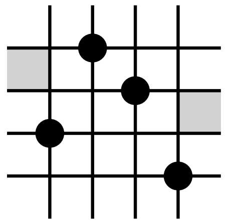
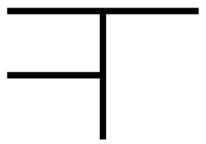
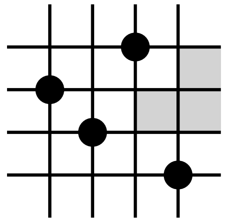
		w	$1, 2, 6, 21, 80, 322, 1347, 5798, 25512, 114236$	A106228*
		s	$1, 2, 6, 23, 102, 499, 2622, 14547, 84229, 504775$	
		w	$1, 2, 6, 21, 80, 322, 1347, 5798, 25512, 114236$	A106228*
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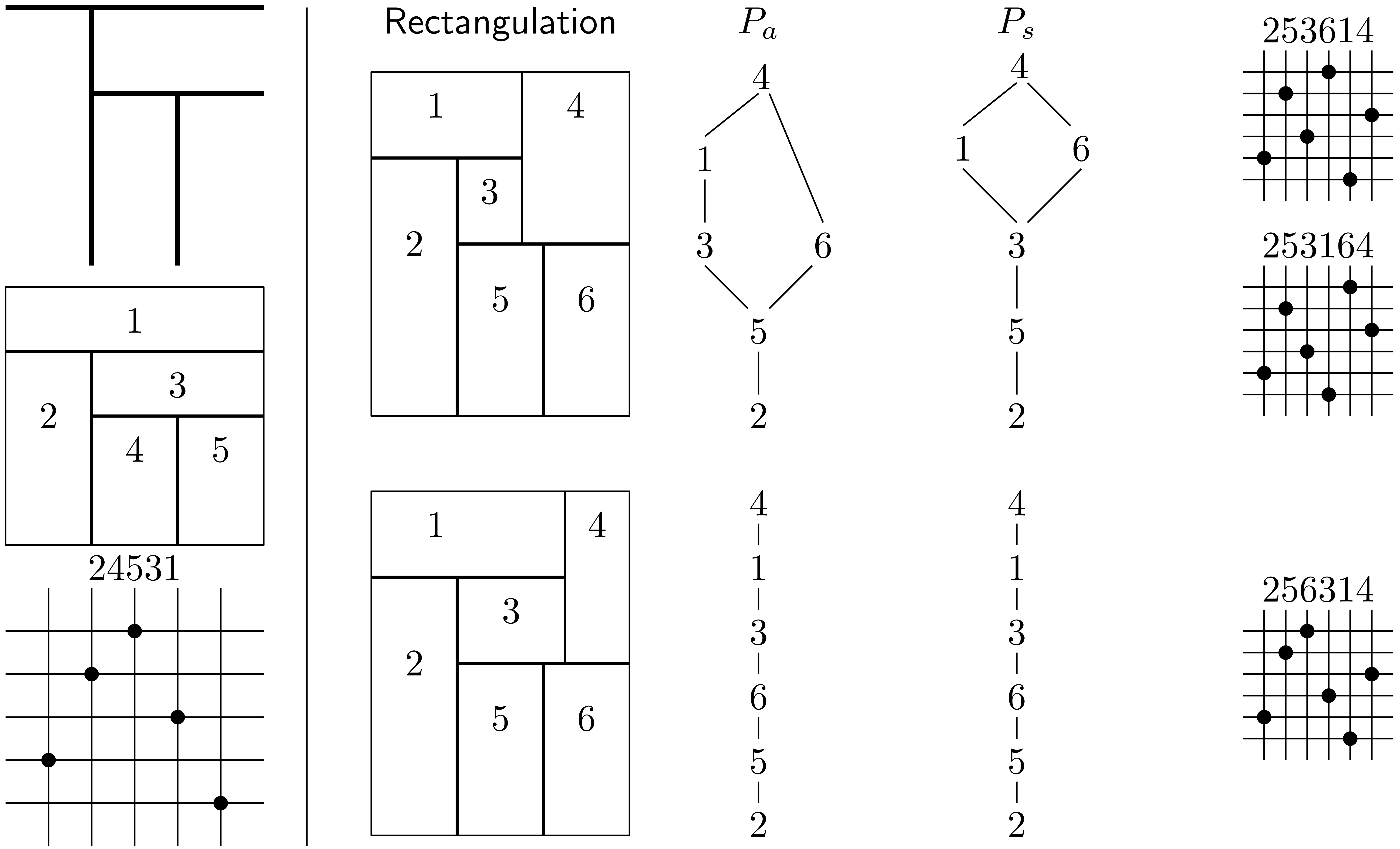
*Conjectured

THANK YOU!

Example of not being able to find mesh pattern



References

Eyal Ackerman, Gill Barequet, and Ron Y. Pinter. “A bijection between permutations and floorplans, and its applications.” Discrete Applied Mathematics, 154 (2006), 1674–1684.

Andrei Asinowski and Cyril Banderier. “From geometry to generating functions: rectangulations and permutations.” Séminaire Lotharingien de Combinatoire, 91B.46 (2024).

Andrei Asinowski, Jean Cardinal, Stefan Felsner, and Éric Fusy. “Combinatorics of rectangulations: Old and new bijections.” arXiv:2402.01483 (2024).

Stefan Felsner, Éric Fusy, Marc Noy, and David Orden. “Bijections for Baxter families and related objects.” Journal of Combinatorial Theory, Series A, 118:3 (2011), 993–1020.

Shirley Law and Nathan Reading. “The Hopf algebra of diagonal rectangulations.” Journal of Combinatorial Theory, Series A, 119:3 (2012), 788–824.

Arturo Merino and Torsten Mütze. “Combinatorial generation via permutation languages. III. Rectangulations.” Discrete and Computational Geometry, 70 (2023), 51–122.

Nathan Reading. “Generic rectangulations.” European Journal of Combinatorics, 33 (2012), 610–623