Scottish Combinatorics Meeting 2024

University of St Andrews

27 May – 28 May 2024

1 Invited Talks

Mary Cryan (University of Edinburgh)

Euler-tours of low-height toroidal grids

The problem of exactly counting the Euler tours (ETs) of an (undirected) 4-regular graph is known to be #P-complete, and to date no fpras exists for approximate counting. The natural "Kotzig moves" Markov chain converges to the uniform distribution on Euler tours of the given graph, but attempts to show rapid mixing (even for restricted classes of graphs) have been unsuccessful. For the specific case of a toroidal grid with a constant number of rows k, a transfer matrix can be defined and used to exactly count Euler tours of that grid. We show that we can use some of that same structure to prove rapid mixing of the Kotzig moves chain on 2-rowed toroidal grids, and discuss the issues for higher number of rows. (Joint work with Sophia Jones. I will touch on the details of the transfer matrix method for ETs, which was joint with Creed, Astefanoaei, and Marinov).

Felix Fischer (Queen Mary University of London)

Optimal Impartial Selection

I will look at selection rules that map any directed graph (without loops) to a subset of its vertices. A selection rule is impartial if the selection of a vertex is independent of its outgoing edges, and an optimal impartial rule is one that in any graph selects vertices with large indegrees. The idea is that vertices and edges represent individuals and nominations among individuals, and that we want to select highly nominated individuals without individuals having an influence on their own selection. I will discuss what is know about impartial selection and point to some open problems. The talk is based on joint work with Noga Alon, Antje Bjelde, Javier Cembrano, David Hannon, Max Klimm, Ariel Procaccia, and Moshe Tennenholtz.

Nora Frankl (The Open University)

Monochromatic infinite sets in Minkowski spaces

By a result of Erdős, Graham, Montgomery, Rothschild, Spencer, and Straus it is possible to colour the points of the d-dimensional Euclidean space with two colours such that there is no monochromatic isometric copy of a given infinite set. We prove several related results in other normed spaces. We show that the same is true in 2-dimensional l_p spaces. We also prove that if the unit ball of a normed plane is a polygon, then two colours are not enough. Further, we also show an example of an infinite set for which we need to colour with an arbitrarily large number of colours to avoid its monochromatic isometric copies in a space with the max norm of sufficiently large dimension. Joint work with Panna Gehér, Andrey Kupavskii, Arsenii Sagdeev and Géza Tóth.

Sergey Kitaev (University of Strathclyde)

Singleton mesh patterns in multidimensional permutations

Permutation patterns is a popular area of research introduced in 1968, but with roots going to the work of Leonhard Euler in 1749. In this talk, I will present a brand-new notion of a singleton mesh pattern (SMP), which is a multidimensional mesh pattern of length 1. It turns out that avoidance of this pattern in arbitrary large multi-dimensional permutations can be characterised using an invariant of a pattern called its rank. This allows to determine avoidability for an SMP P efficiently, even though determining rank of P is an NP-complete problem. Moreover, using the notion of a minus-antipodal pattern, one can characterise SMPs which occur at most once in any d-dimensional permutation. I will also discuss a number of enumerative results regarding the distributions of certain general projective, plus-antipodal, minus-antipodal and hyperplane SMPs. This is joint work with Sergey Avgustinovich, Jeffrey Liese, Vladimir Potapov and Anna Taranenko.

Kitty Meeks (University of Glasgow)

The search for useful temporal graph parameters

A highly successful approach for tackling NP-hard problems on graphs is the paradigm of parameterised complexity: the running time of algorithms is analysed in terms of a secondary measure, or parameter, in addition to the total input size, and the goal is to restrict the combinatorial explosion to the parameter (which is hopefully, in instances of interest, much smaller than the total input size). Many widely used parameters capture structural properties of the input graph, for example the edit distance to some class on which the problem is tractable, or the ease with which the graph can be decomposed according to specific rules. In recent years, there have been numerous attempts to apply the parameterised approach to algorithmic problems on temporal graphs, in which the edge-set changes over time. However, this has led to efficient parameterised algorithms in only a few cases: for many natural problems (including some that are polynomial-time solvable on static graphs) the additional complexity introduced by encoding temporal information in the graph renders the temporal versions intractable even when the underlying graph has very simple structure (e.g. when it is a tree or even a path). This motivates the search for new parameters, specifically designed for temporal graphs, which capture properties of the temporal information in addition to the structure of the underlying graph. In this talk I will survey recent progress in the development of such parameters and highlight a large number of remaining challenges; in the process I will describe joint work involving Benjamin Bumpus (U Florida), Jessica Enright (University of Glasgow), Laura Larios-Jones (University of Glasgow) and Samuel Hand (University of Glasgow).

Siaw-Lynn Ng (Royal Holloway University of London)

Some connections between finite geometry and applications in information security

Finite geometry and combinatorics has been very useful in modelling and benchmarking many problems arising from digital communications, information security and cryptography, by facilitating the study of the essential structures and their interactions. While using existing combinatorial objects to provide constructions and benchmarks for various applications has been productive, more in-depth consideration of whether the properties of these objects are strictly what is required in the applications can also yield interesting insights, and some applications can also provide inspirations and results in other areas. I would like to talk about some structures in finite geometry arising from some applications in information security, with the aim that we might come to a better understanding of their properties and their relationships to existing structures, and that this might give us further avenues of research.

John Sheekey (University College Dublin)

Finite semifields and their combinatorial applications

Finite semifields are non-associative division algebras. They have been studied since the early 1900s for a variety of reasons, including connections to finite geometry and combinatorics. A classical example of this is through their use in constructing projective planes, where one directly replaces the use of a field with a semifield. However they also lead to interesting geometric and combinatorial objects in more surprising ways. In this talk we will give an introduction to the topic, and highlight a few of these unconventional applications through some recent results.

Jason Smith (Nottingham Trent University)

Linear Extensions of Posets and an application to Neuroscience

Pairs of neurons that both send and receive information from each other are conjectured to play an important role in the reliability of the brain. The brain can be modelled as a directed graph, where neurons are vertices and edges are synapses, so such connections are bidirectional edges. Computations indicate that that these bidirectional edges occur more frequently within cliques in brain networks. However, bidirectional edges naturally create more cliques. So to understand whether this prevalence of bidirectional edges in cliques is expected or abnormal we must understand how many cliques we would expect from some given bidirectional edges. We show that this problem is equivalent to counting the number of linear extensions of a poset, and counting the number of permutations with a given inversions set. And we present some new formulas for the number of linear extensions for particular classes of posets using modular decomposition.

2 Contributed Talks

Raad Al Kohli (University of St Andrews)

On classes of groups defined by formal language classes

We introduce a a family of finitely generated groups $EpiC$ where C is a language class that is closed under homomorphisms. We particularly focus on the class of epiregular groups. A group G with a finite generating set X is epiregular if there is a regular language R over X such that

1. there is no word in R that represents the trivial element of G, and
2. for every non-trivial element g in G, there is a representative w in R of g.

Some of the results we shall discuss are as follows. Epiregularity is independent of generating set. There are uncountably many finitely generated groups which are not epiregular. We shall also discuss how this class of groups is related to other well-studied classes of groups defined by formal languages, such as context-free groups and $coCF$ groups. Further, we show this class of groups does not coincide with other previously studied classes of groups. We also discuss properties some closure properties of this class of groups.

Noura Alshammari (University of Strathclyde)

Asymptotic enumeration of monotone permutation grid classes

A monotone grid class Grid(M) is a set of permutations whose shape satisfies constraints specifies by the matrix M, all of whose entries are in $\{1, -1, 0\}$. Each entry of the matrix corresponds to a cell in a gridding of a permutation. If the entry is 1, then the points in the cell must increase.

If it is -1, they must decrease. If it is 0, the cell must be empty. To find the exact number of permutations of length n in Grid(M) is hard, because a permutation may have more than one gridding, so we only determine the asymptotic enumeration. To do this, we find the number of gridded permutations, the typical proportion of points in each cell, and the ways in which permutations can be gridded. Our focus is mainly on L-shaped, T-shaped, and X-shaped classes.

Herman Chen (Chongqing Normal University)

Representing split graphs by words

There is a long line of research in the literature dedicated to word-representable graphs, which generalize several important classes of graphs. A number of recent papers are dedicated to the study of word-representability of split graphs, an important class of graphs. A split graph is a graph in which vertices can be partitioned into a clique and an independent set.

In this talk, I will discuss several results on word-representability of split graphs including that threshold graphs, a subclass of split graphs, are word-representable. Further, I will mention several general theorems on word-representable split graphs that are used to characterize computationally such graphs with cliques of size 5 in terms of nine forbidden subgraphs, thus extending the known characterization for word-representable split graphs with cliques of size 4. Moreover, I will discuss how to use split graphs to show that gluing two word-representable graphs in any clique of size at least 2 may, or may not, result in a word-representable graph, which answers a question that was open for about ten years.

Val Gladkova (University of Cambridge)

A Lower Bound for the Strong Arithmetic Regularity Lemma

The strong regularity lemma is a combinatorial tool originally introduced by Alon, Fischer, Krivelevich, and Szegedy in order to prove an induced removal lemma for graphs. Conlon and Fox showed that for some graphs, the strong regularity lemma must produce partitions of wowzer-size. In this talk, we will sketch a proof showing that a comparable lower bound must hold for the arithmetic analogue of this lemma, in the setting of vector spaces over finite fields.

Frederik Glitzner (University of Glasgow)

Structure and Fairness of Stable Partitions

The Stable Roommates problem is a classical combinatorial problem with applications to computational social choice. Imagine a group of friends who want to play tennis with each other, where everyone has preferences over who to play with. Can we match them into pairs such that no two friends prefer to play with each other rather than their assigned partners? While some problem instances admit such stable matchings, others are unsolvable and do not.

In this talk, we will discuss the existence and structure of stable matchings and consider stable partitions (also called stable half-matchings) as a generalised solution concept. Contrary to stable matchings, stable partitions are known to always exist. We will extend the study of stable partitions, further characterise their structural properties, and demonstrate how to efficiently enumerate all of them. However, listing them all might not be feasible in practice. Therefore, we identify tractable and approximable variants of finding fair stable partitions directly by adapting common optimality measures from stable matchings.

Humaira Hameed (University of Strathclyde)

On semi-transitivity of (extended) Mycielski graphs

An orientation of a graph is semi-transitive if it is acyclic, and for any directed path $v_0 \to v_1 \to \cdots \to v_k$ either there is no arc between v_0 and v_k, or $v_i \to v_j$ is an arc for all $0 \leq i < j \leq k$. An undirected graph is semi-transitive if it admits a semi-transitive orientation. Semi-transitive graphs generalize several important classes of graphs, and they are precisely the class of word-representable graphs studied extensively in the literature.

The Mycielski graph of an undirected graph is a larger graph, constructed in a certain way, that maintains the property of being triangle-free but enlarges the chromatic number. These graphs are important as they allow to prove the existence of triangle-free graphs with arbitrarily large chromatic number. An extended Mycielski graph is a certain natural extension of Mycielski graphs.

Victoria Ironmonger (University of St Andrews)

Well quasi-order for equivalence relations and other structures under consecutive orders

A poset is well quasi-ordered (wqo) if it contains no infinite antichains or infinite descending chains; this property gives a way of distinguishing between those posets which are 'tame' and 'wild'. We consider posets where combinatorial structures are related when one is a substructure of the other. Given such a poset, avoidance sets are subsets which are defined by their forbidden substructures. The wqo problem asks if it is decidable, given a finite set, whether its avoidance set is wqo. We will discuss this problem for various structures under consecutive orders, and outline the proof of decidability for equivalence relations. This involves adapting methods used by McDevitt and Ruškuc for permutations and words, which associate structures with paths in certain digraphs.

Thomas Karam (University of Oxford)

Fourier analysis modulo p on the Boolean cube

Fourier analysis in theoretical computer science is most commonly defined on the Boolean cube $\{0,1\}^n$ identified with $\mathbb{Z}_2^n$. Recently, generalisations of that setting have been studied, involving restrictions to $\{0,1\}^n$ of characters modulo p for some arbitrary prime p. Unlike in the case of mod-2 characters, different mod-p characters are no longer orthogonal when restricted to the Boolean cube. We will begin by discussing some of the basic phenomena involved, and then mention some recent results and remaining difficulties.

Laura Larios-Jones (University of Glasgow)

Temporal Reachability Dominating Sets

Given a population with dynamic pairwise connections, we ask if the entire population could be (indirectly) infected by a small group of k initially infected individuals. We formalise this problem as the Temporal Reachability Dominating Set problem on temporal graphs. We provide positive and negative parameterized complexity results in four different parameters: the number k of initially infected, the lifetime τ of the graph, the number of locally earliest edges in the graph, and the treewidth of the footprint graph $\mathcal{G}_{\downarrow}$. We additionally introduce and study the MaxMinTaRDiS problem, where the aim is to schedule connections between individuals so that at least k individuals must be infected for the entire population to become fully infected. We classify three variants of the problem: Strict, Nonstrict, and Happy.

Abigail Ollson (Keele University)

The Insertion Encoding of Cayley Permutations

A Cayley permutation is a sequence of numbers 1 to n such that every number appears at least once; for example 112 is a Cayley permutation of size three. If every number appears exactly once in a Cayley permutation, then it is precisely a permutation. Cayley permutations are counted by the Bell numbers as they are in bijection with ordered set partitions; the value i in the jth block of the ordered set partition corresponds to the ith index of the Cayley permutation having value j. A Cayley permutation π contains another Cayley permutation σ if there is a subsequence of π that is order isomorphic to σ, not necessarily at consecutive indices. For example, in 123413 the subsequence 3413 implies an occurrence of 2312. Sets of Cayley permutations that are closed downwards with respect to pattern containment are called Cayley permutation classes and can be defined by the set of minimal Cayley permutations not in the class, called the basis. For a basis B, we denote the class of Cayley permutations avoiding B as $Av(B)$.

The insertion encoding is a language that encodes how to construct permutations by adding new maximums that was introduced by Albert, Linton and Ruskuc [1]. We extend this to Cayley permutations. As it is possible to have multiple maximum elements in a Cayley permutation, we specify when the value being added is a new maximum or a repeated element, and prioritise the rightmost maximum.

When the insertion encoding for a Cayley permutation class is a regular language, we call the class regular. We give a condition on the basis of a class for the class to be regular, and by using the insertion encoding, we determine an algorithm to compute this regular language. This includes $Av(12)$, $Av(231, 312, 121)$ and $Av(1211, 1234, 3421)$, for example. We also compute the regular language for $Av(231, 312, 1212)$, a class studied by Cerbai [2] called the hare pop-stack sortable Cayley permutations.

[1] Michael Albert, Steve Linton, and Nikola Ruskuc. The insertion encoding of permutations. Electr. J. Comb., 12, 09 2005.

[2] Giulio Cerbai. Sorting cayley permutations with pattern-avoiding machines. Australian Journal of Combinatorics, 80:322-341, 2021.

James Walrad (University College London)

Stieltjes moment sequences with support in $[\xi, \infty)$

Many integer sequences arising in enumerative combinatorics are Stieltjes moment sequences, (i.e., the moments of a positive measure on $[0, \infty)$). Stieltjes showed in 1894 that a sequence of real numbers is a Stieltjes moment sequence if and only if its ordinary generating function can be written as a continued fraction of Stieltjes type with nonnegative coefficients. If we demand that the representing measure be supported in some fixed closed subset $K \subseteq [0, \infty)$, what constraints does it put on the continued fraction coefficients? Wall solved this problem in 1940 for the case $K = [0, \xi]$. Here we solve it for the case $K = [\xi, \infty)$. The arguments have an algebraic/combinatorial flavor. Joint work with Alan Sokal (UCL, NYU).